

Oil and Gas Depletion: Diffusion Models and Forecasting under Strategic Intervention

Esaurimento del Petrolio e del Gas: Modelli di Diffusione e Previsioni in Presenza di Interventi Strategici

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Riassunto: L'esaurimento del petrolio e del gas naturale può essere descritto mediante un processo di diffusione a ciclo di vita finito. Qui si propone un modello generalizzato di Bass, GBM. L'adozione è motivata dall'ipotesi che la diffusione delle tecnologie basate su tali fonti energetiche possa spiegare le correlate produzioni di combustibili in presenza di interventi regolatori. L'analisi statistica è basata su metodologie non lineari atte ad esplorare aspetti predittivi e strategici.

Keywords: generalized Bass model, oil peak, gas peak, nonlinear regression.

1. Introduction

The Council of Foreign Relations of United States of America and the James A. Baker III Institute for Public Policy of Rice University sponsored, in the year 2000, an independent task force on energy policy. E.L. Morse (chair) and A.M. Jaffe (project director) had a pivotal role during the research work and published in spring 2001 an interesting and realistic report "Strategic Energy Policy Challenges for the 21st Century". The analyses and the recommendations of the report of Morse and Jaffe (2001) gave new light in political and economic scenarios. It is well-known that the lower 48-States and Alaska have practically produced all discovered petroleum resources.

The U.S.A has been without an energy policy since 1940. This means that no attention has been paid to the central role of this policy for the economy and for security. It appears that there are no easy and quick solutions to today's energy problems, but only hard policy tradeoffs between competing interests. Until now, strong worldwide economic growth has been sustained by surplus capacity in hydrocarbon fuels. New global demands for more energy are facing the beginning of capacity limitations so that the U.S.A. can no longer assume an increasing oil production. Furthermore, dependence on only a few foreign sources is strategically and politically wrong. Emerging technologies are not yet commercially viable to fill shortage and will not be for some time so that during the next decades there is a high probability of an oil-supply crisis.

Let us consider now some historical references. Many years ago Hubbert (1949) made his first prediction that the fossil fuels era would have a very short duration. In 1956 he estimated that U.S.A. oil production would peak around 1970 and this estimate in fact turned out to be accurate. Recent geological theories and empirical studies strongly confirm that a world oil peak is imminent. Campbell and Laherrère (1998) published a key-paper on this crucial problem. There are also many new contributions in this area which extend

and study the demographical and economical implications of a fuel resources depletion (see Laherrère (2003)). Many geological estimations based upon a technical evaluation of reserves are not very reliable because the resources have been over-estimated for political and financial reasons. A solution could be a model-building theory based upon instantaneous or cumulative production data.

In this note we consider oil and gas production as a diffusion model. In particular, we refer to a Generalized Bass Model (GBM) introduced by Bass *et al.* (1994) and examined in Guseo (2002). Our purpose is to include in the standard version of the model, which is a special specification of the Riccati equation, an intervention variable useful for separating stochastic disturbances from modified systematic life-cycle behaviour.

2. Generalized Bass Model Rationale

The basic assumption of GBM relies on a simple partition of instantaneous “adoptions” into two different sets. The first block refers to “innovators” and is defined via a direct proportion p of the residual market, $p(m - z)$, where parameter m describes the total unknown market and z is the cumulative function of adoptions over time. The second block is characterized by a delay in adoption due to a word-of-mouth effect. A way to represent this effect is based upon a penalized access to the residual market, via a simple ratio z/m , modeled by a parameter q characterizing “imitators”, $qz(m - z)/m$. Bass *et al.* (1994) consider a very general perturbation of the standard Bass model by an integrable function $x(t)$ which may reasonably oscillate around 1. If $x(t) = 1$ we have the standard Bass model, which is a special case of the well-known Riccati equation; see Guseo (2002). The generalized Bass differential equation is therefore the perturbed sum of the previous two components,

$$z' = m \left(p + q \frac{z}{m} \right) \left(1 - \frac{z}{m} \right) x(t), \quad (1)$$

and the general closed form solution is

$$z(t) = m \frac{1 - e^{-(p+q) \int_0^t x(\tau) d\tau}}{1 + \frac{q}{p} e^{-(p+q) \int_0^t x(\tau) d\tau}}. \quad (2)$$

We may consider oil and gas as a production directly controlled by correlated diffusion processes, i.e., individual mobility, gas and oil heating, cooling and air conditioning, industrial applications, etc.. As we see later, in this area “imitation” effects have a dominant role.

Statistical implementation of such a model may consider two main features. The first one models the “intervention” due to political or economical changes of regime (local changepoints), and these can be read as “deterministic” effects.

A simple and parsimonious parametric version of the intervention function $x(t)$ may firstly take the form with two exponential shocks,

$$x(t) = 1 + c_1 e^{b_1(t-a_1)} I_{t \geq a_1} + c_2 e^{b_2(t-a_2)} I_{t \geq a_2}, \quad (3)$$

and a second one with two rectangular shocks,

$$x(t) = 1 + c_1 I_{t \geq d_1} I_{t \leq h_1} + c_2 I_{t \geq d_2} I_{t \leq h_2}. \quad (4)$$

The second feature must model all the “micro changes” governed by multiple stochastic effects. A simple specification of a statistical model may assume a nonlinear regressive nature, i.e., $y(t) = z(t) + \varepsilon$, where ε is an i.i.d.residual error component. In some cases ε may be normal distributed, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. Statistical inference and computational aspects under such assumptions have reached a well–founded basis during the last two decades, Seber and Wild (1989).

3. World Oil and Gas Depletion Models

Quality of oil and gas data may be sufficiently reliable with reference to production data. Here we consider daily world production in thousands of barrels. The data sources are Industriedatenbank (2001) and Energy Information Administration (2001). Data cover a century: from 1900 to 2001. Information is essentially based on crude oil data with a limited contribution of NGL (natural gas liquids) during the last decade. Statistical identification of the model with exponential shocks is performed on cumulative data and the analysis is satisfactory. The estimation of coefficients points out the significative effect of the parameters a_1, b_1, c_1 linked to the second oil crisis which occurred around 1979. On the contrary, a model with possibly two shocks, highlights the statistically negligible effect of the first oil crisis that occurred in 1973.

Estimation method: Marquardt

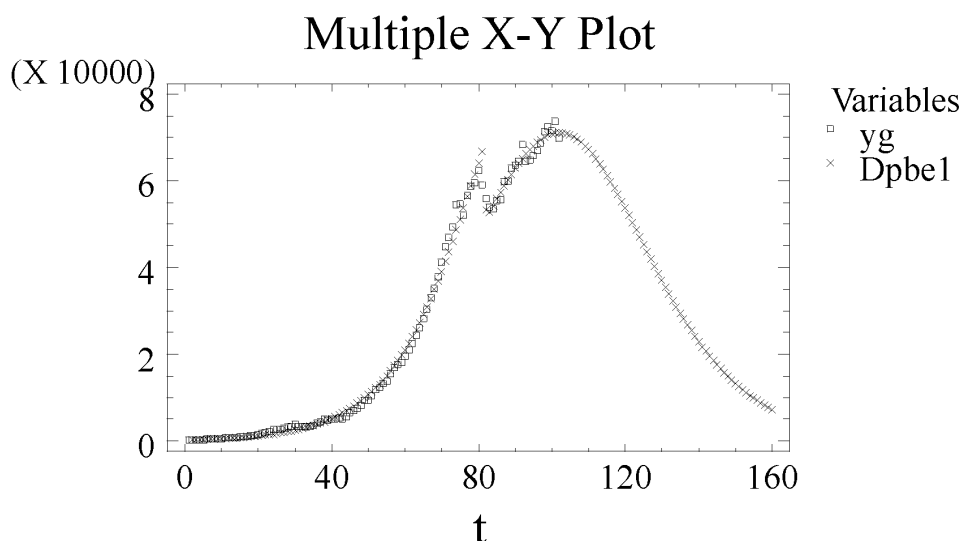
Parameter	Estimate	Asymptotic Standard Error	Asympt. 95% Conf. Interval	
			Lower	Upper
m	4,8127E6	361342,0	4,09544E6	5,52995E6
p	0,0000563914	0,00000255151	0,0000513267	0,0000614561
q	0,0752259	0,000875335	0,0734884	0,0769634
c1	-0,259008	0,0140402	-0,286878	-0,231139
b1	-0,00918896	0,0122075	-0,0334208	0,0150428
a1	81,1114	0,388108	80,341	81,8818

Analysis of Variance				
Source	Sum of Squares	Df	Mean Square	
Model	8,8534E13	6	1,47557E13	
Residual	4,37617E9	96	4,55851E7	
Total	8,85384E13	102		

R-Squared = 99,9921 percent
Standard Error of Est. = 6751,67
Durbin-Watson statistic = 0,0572554

Some statistical improvements may be obtained by considering: a) simultaneous exact confidence regions (see, for instance, Guseo (1983), Bates and Watts (1988)) in order to consider intrinsic curvature aspects of solution locus; b) weighting criteria depending upon heteroskedastic dispersion proportional to instantaneous effects, $\sigma^2(t) = \sigma^2 z'(t)/m$; c) an autoregressive structure of residuals. Here we skip these aspects in order to simplify matters. The main results determine an ultimate recoverable resource (URR) of 1757 Gbo (Giga barrels of oil, $m \cdot 365$), a world oil peak in year 2003 and a 90% depletion around year 2034. Plot in Figure (1) represents daily production (a box), in thousands of barrels per year with the origin being 1900, and estimated values (a cross). Similar results are attained, with BP (British Petroleum) data on natural gas, via a GBM with a rectangular shock. In this case the world gas peak is estimated to occur around the year 2013 and a 90% resource depletion is forecast to occur around 2055.

Figure 1: Oil depletion with GBM under an exponential shock (around 1979).



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