

World Oil Depletion Models: Price Effects Compared with Strategic or Technological Interventions

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Abstract

World oil depletion, including natural gas liquids, have been modeled recently in [18] and in [14] following life cycle diffusion models. We examine joint effects of strategic, economical and technological interventions within Generalized Bass Models (GBM). Statistical analysis takes into account three different hierarchical levels: natural diffusion, long memory interventions and stochastic components. The main results confirm historical 1970's shocks and, more specifically, the strong long memory effect in production after World War II. The estimated peak-date, 2007, and the 95% depletion time, 2023, are determined under an equilibrium intervention hypothesis.

Key words: oil peak, depletion times, oil price inelasticity, generalized Bass model, nonlinear models, armax

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1 Introduction

One of the nowadays main problems is the development of a coherent and feasible energy policy for economy and security. It is interesting to read the answers to a dozen of questions on the role of science and technology of the two candidates at the President of USA position, as reported by *Nature* in september 2004. Both candidates, Bush and Kerry, avoid to treat in detail this problem which is certainly crucial but not useful for election result. It is probable that both would agree on that only hard policy tradeoffs between competing interests may be the unique road-map.

The powerful worldwide economic growth after the World War II has been sustained by surplus capacity in hydrocarbon fuels, in particular by crude oil intensive production, (see Figure 1). Emerging economies such as China, India and other countries may increase the new global demand for more energy. China is strongly engaged in coal extraction in order to sustain the internal electric increasing demand of a day by day expanding economy.

The demographic effects of the too rapid economic growth are well described in [12]. The trend toward urbanization is clear. The rural population of the rich countries peaked around 1950 and slowly declined since then. World average length of life raised from about 30 years at the beginning of 20th century to 65 years at the beginning of 21st century. Current global population growth rates are far higher than any experimented before World War II. Nowadays world population of 6.3 billions will peak, around 2050, till 8.9 billions (medium variant scenario of United Nations Population Division, World Population Prospects: the 2002 Revision, Highlights). At the same time, thirty among

the more developed countries are expected to have a lower population in 2050 than today (Japan -24% , Italy -22% , FSU -29%). Migrations, food, housing, education, health, employment and public order pose formidable challenges to economy, and to social and political governance.

The same topic was examined in the past by Marchetti (see e.g. [25], [26], [27]). He highlighted that *niche* of mankind is expanding under the new advances of technology even if diffusion of population is not homogeneous, especially in Europe. Nevertheless, some regularities in the evolution of inventions and innovations in waves, supported by new energy resources, are strongly promising and opposite to Malthusian approaches. In [27] information, knowledge and science are the new paradigm of *Logos*.

During World War II, USA dominance in production and refining of petroleum provided a decisive military advantage (see [28]). USA supplied the 90% of Allied Forces. Infrastructure development accelerated demand. New Jersey high speed and limited access roadways, Holiday Inns highway motels, fast-food and drive-in are worldwide known symbols of the 1950's.

FIGURE 1 ABOUT HERE

Technology has generally led to a greater use of hydrocarbons fuels for most human activities making civilization vulnerable to decreases in supply as it is well highlighted by [19].

The most expanded economies are facing the beginning of capacity limitations so that the assumption of an increasing oil production in the next decade does not seem realistic. Emerging technologies are not yet commercially or technologically viable to fill completely the shortage even if some interesting hopeful

perspectives emerge, see for instance [26] and [14]. The strong economic growth after the World War II seems to be the foundational basis of a structural modification of global economy and correlated energy consumptions.

It is well-known that "technical" evaluation of reserves is not very reliable because the resources have been over-estimated for financial reasons. To overcome this drawback and the problems concerning the limited flexibility of the standard Hubbert model, a possible solution has been proposed in [17] and [18] and is linked to a model-building theory based upon cumulative production data with special controlling functions.

In this paper we extend [18] and [14] results by considering oil production as a diffusion model and we refer to special generalized Bass models (GBM) firstly introduced in [6]. Our purpose is to include intervention variables useful for separating stochastic disturbances from systematic changes in life-cycle behaviour possibly due to strategic effects, price effects, etc.. In Section 2, we briefly introduce the proposed diffusion Generalized Bass model, including some properties and some useful interpretations. Section 3 deals with some different choices about the intervention variables and compares the derived models w.r. to the stochastic residual component. In Section 4 we test the performances of the models proposed in Section 2 and Section 3 for world oil production forecasting. Section 5 is devoted to the discussion and final comments.

2 Historical background and literature contributions

The pioneer of historical evaluations of crude oil depletion is [20]. In 1956 he

correctly estimated that USA oil production would peak around 1970. Later, in [21], he recognized the explicit role of logistic equation (see [32]) in his previous model. More recent geological studies confirm that a world oil peak is imminent. In particular, Colin J. Campbell, president of ASPO (*Association for the Study of Peak Oil and Gas*), published in 1998 with J.H. Laherrre a key-paper on this topic. From this date in advance many works were developed in this area.

In [29], [29] there are some attempts to include prices and costs in Hubbert model. The most interesting is the second work in which an extension, represented by a linear regressive approach, is based upon a discretization of logistic equation. The emphasis in *non-time-series* is simply misleading, due to the special representation. The main limit is that this approach doesn't give an easy and sure way to evaluate temporal forecasts. Some interesting considerations about inelasticity of demand w.r. to prices are given.

In several papers Laherrère uses logistic equation or other equivalent variations in order to model discoveries, production and population. In [22], [23] there are considerable efforts to apply a multi-Hubbert modelling which we do not consider very clear. The proposed fitting to the more recent part of crude oil production series is misleading because the method erases the memory of old extraction processes with an improper magnification of the residual resources. Statistical analysis of the properties of the applied methods are omitted.

In [4] a class of new models for the representation of a decline in production more abrupt than the growth is provided. Description and analysis are based on a simulated procedure which is not so easy to evaluate with reference, e.g.,

to the persistence of strategies or to an improving of technological factors. Statistical application is not provided.

In [8] a conventional oil modelling by Campbell and Laherrre and the corresponding checking provided by University of Reading is examined. Hubbert model is the basis but with known flaws. Dynamics of Alaska production are not examined. The *technological shock* due to the accelerated extraction of oil from the Prudhoe Bay may be recognized with a more flexible tool and not with a simple logistic model. Very curious is the peak shifting of 10 years to compensate the reduction of demand following the 1970's oil shocks (Yom Kippur war and Iranian revolution). However, there is no mention of a positive shock since 1951 that had strongly modified the basic evolution of oil production dynamics by anticipating the peak and, obviously, contracting depletion times.

In [1] the issue of oscillation of oil prices during the shocks occurred in 1973 and 1980 is addressed. The main result is an unpredictable relationship between demand and supply. Evidence is given to a common effect of that shocks, i.e., a transition to low-energy tools (computers, electronics, robotics, bio-engineering, etc.) as compared with traditional industrial products, steel-car, chemicals, etc.. The role of the "well-informed spectators" of the oil game, i.e., journals, specialized consultants, institutes, is important as they "massage the message" and, clearly, influence oil price-settings. This result contributes to enhance our work which highlights the surprising weak relationship between observed prices, oil demand and supply. The decade post 1986 is relatively stable with some exceptions that confirm paradoxical behaviours of the quantity-price relationship.

In [2] the main results of WOCAP model with a world oil peak estimated around 2006–07 and an associated production likely by 81 mb/d is described. The model, developed upon oil reserve estimates by Colin J. Campbell, predicts global production of conventional oil for all the hydrocarbon liquids, such as NGL, etc.. Estimation of daily production of 2020 is 55 mb/d.

In [9] the significance of a structural change in the Pindyck model is tested. Dynamics of oil prices are based upon an extension of an AR(1) process (a structural ARMAX). The nested to the boundaries fixed coefficients sub-model is proved to be sufficient even if no deeper analysis is performed. A raw control reveals that the reduced model gives rise to a low level for R^2 , namely $R^2 = 0.81$. It is surprising this use of very complex models that, perhaps, do not gain significance.

The nature of forecasting errors in this area is discussed in [24] w.r. to geophysical models and, in particular, to Hubbert model. Obviously, pure Hubbert models have no room for economics since the basic differential equation is the classical logistic. The under-estimation of Hubbert model is a known aspect but no analysis is given in order to introduce more flexible tools capable, for instance, to track an intervention or a systematic perturbation. Lynch supposes that the origin of the problem relies on the strong assumption of a static URR (Ultimate Recoverable Resource). The impact of technology in expanding resources is sometimes reasonable but a lot of systematic interventions may affect normal evolution. For instance, the forced production of Prudhoe Bay in Alaska was a positive shock, or, on the contrary, the negative effect of accidents in pipelines (Piper Alpha disaster) in the North Sea depressed regular production. These accidents led to examine restructuring of UK offshore safety legislation followed by major changes to the Petroleum Revenue Tax.

These radical interventions are not recognized by a simple Hubbert model. A solution to these claims is discussed in [18].

An attempt to introduce, among others, elements of an economic theory for oil supply is proposed in [3]. The paper is quite unusual because scientific approach to analysis and discussion is systematically omitted in favour of a caustic speech against Hotelling's ideas and other people, Lewis, Friedman, Gylfason, Solow among others. Emphasis is "on getting the economics right, and getting economics right is not always a prelude to getting the forecast right – especially in the oil market". This is a reductive idea of modelling which excludes the empirical corroboration or falsification, following a popperian approach that establishes a well-known boundary to the characterization of scientific theories. Nevertheless, a lot of linguistic effort is spent to compare diverse differential models, e.g., exponential law and logistic–Hubbert law. The long discussion about the R/q ratio (reserves vs. production) may be simplified because the equation $R/q = \tilde{c}$ is equivalent to a monomolecular model. The "false impression of the availability of oil" is not demonstrated. To do this, a simple comparison between monomolecular and logistic equations explains the issue in a straightforward manner. Some referred applications of Hubbert model to UK North Sea and Norway are not cited to adequately identify the sources. "Pressure" analysis in the oil deposits, in order to explain the role of Hubbert model, is quite interesting but this suggestion is not really translated in an extended model. However, a good argument is given by the example of Russian strategies vs. OPEC's strategies. High prices of oil are essential for the recovery of Russian economy and have generated an increase of oil export (USA and UE). An acceleration on extractions may reduce the "pressure" on deposits reducing the capacity of the source. How to model this effect in order

to test its significance is not performed. The final comments are contradictory, e.g., "muscle" preventing actions or interventions to postpone the peak and the evaluation of the ability of Hubbert model to describe oil production evolution of the entire world. These theoretical uncertainties reveal that Banks is fully aware that Hubbert model is too simplistic. An interesting comment refers to the share market boom of 1990's. It is attributable to a low inflation rate as a result of a long period of low oil prices. If this is true, increasing oil prices may determine a high inflation rate as an automatic reaction to depletion. This simple deduction does not consider, however, compensating effects due to possible substitutions.

3 GBM diffusion model

A diffusion process may be described with different emphasis on moment components. In stochastic differential approach the main effort is devoted to the second order aspect of diffusion. Here, the main interest is based upon the first order moment of a process because of its possible non stationarity due to initializing aspects, monotonicity and asymptotic character of the saturating effects of a life-cycle pattern.

The main ideas on this topic are based on models developed in [5], and in [6], where the latter work extends the previous one by connecting the diffusion to external intervention factors. Both models may be thought as special cases of Riccati equation which, in turn, includes the logistic equation.

The diffusion of an innovation in a social system may depend upon two principal forces which describe the asymmetric information of the agents in a system

or in an economy. Bass simplified the complexity of the situation by considering, at a first step, only two types of agents. The *innovators*, that give direct attention to advertising or communication of companies. The *imitators*, that adopt the innovation only in a second time by reinforcing a personal opinion on the basis of a word-of-mouth effect.

A Bass model describes the life-cycle of a generic product by the cumulative function $z = z(t)$. It is a function of time and potential market, m , (*carrying capacity*). On the basis of earlier historical data of diffusion, it is possible, with different degrees of confidence, to estimate the diffusion constants and, in particular, the carrying capacity. The misleading or truly false information regarding over-accounting of crude oil reserves in financial transactions is fully ignored.

Let us denote by z' the instantaneous adoptions that are decomposed into two additive parts. The first one, $p(m - z)$, is referred to innovators which adopt with a constant rate p over time. The difference $(m - z)$ depicts the residual market. The second component is characterized by a delay in adoptions due to a word-of-mouth effect expressed by a simple ratio z/m which modulate the accessibility of imitators to the residual market, i.e., $q(z/m)(m - z)$.

In [6] a very general perturbation of the standard Bass model by an integrable function $x(t)$ is proposed. This function can vary around 1 and represents political, economical and structural interventions. If $x(t) = 1$ we attain the standard Bass model, BM.

The generalized Bass model, GBM, is therefore

$$z' = m \left(p + q \frac{z}{m} \right) \left(1 - \frac{z}{m} \right) x(t) = \left(p + q \frac{z}{m} \right) (m - z) x(t), \quad (1)$$

and the general closed form solution is

$$z(t) = m \frac{1 - e^{-(p+q) \int_0^t x(\tau) d\tau}}{1 + \frac{q}{p} e^{-(p+q) \int_0^t x(\tau) d\tau}} = mF(t), \quad 0 \leq t < +\infty. \quad (2)$$

The function $F(t)$ represents the modified Riccati distribution function, see [16]. The intervention function $x(t)$ allows a local modified perception of the residual market, $(m-z)x(t)$. In other words, the intervention function modifies the geometry of time, and not the carrying capacity, m , and the intrinsic diffusion parameters p and q . This is very important in many fields. It may be easily proved, see [17], that asymptotic quotas of innovators and imitators are not affected by $x(t)$. The modifications of $x(t)$ are effective during the central part of the cycle.

4 Statistical modelling of GBM: hierarchy of components

In the sequel we may consider oil as a production directly controlled by correlated diffusion processes, i.e., individual mobility, heating, electricity production, chemical applications, etc., so that crude oil production may be thought as a parallel diffusion process. Such production is influenced by regulatory or strategic interventions of political and economical nature which are usually persistent and conceptually deterministic.

A simple parametric description of the intervention function $x(t)$ in Equation (1) may be modeled by some exponential shocks, for example three,

$$x(t) = 1 + c_1 e^{b_1(t-a_1)} I_{t \geq a_1} + c_2 e^{b_2(t-a_2)} I_{t \geq a_2} + c_3 e^{b_3(t-a_3)} I_{t \geq a_3}, \quad (3)$$

if we consider a non uniform distribution of memory effects on intervention.

The coefficients c_i , $i = 1, 2, 3$, control depth and sign of perturbation, the coefficients b_i , $i = 1, 2, 3$, describe the nature of persistency of the effect: usually they are negative if the memory of effect is decaying to the stationary position (mean reverting), i.e., $x(t) = 1$. Sometimes, the coefficients b_i , $i = 1, 2, 3$, may be positive and this aspect introduces a strong acceleration in the saturation of the life-cycle. The coefficients a_i , $i = 1, 2, 3$, denote the starting times of exponential shocks.

The common intervention tool $x(t)$ may be defined with special interest to implementation of exogenous variables, i.e., prices variation or other impact variables in order to test their effects, if any.

Local diffusions may also be anticipated or delayed for a lot of reasons and/or micro-decisions so that it is natural to include stochastic components in order to model them.

A simple specification of a statistical version of a GBM model may be of nonlinear regressive nature, i.e.,

$$y(t) = f(\beta, t) + \varepsilon(t), \quad (4)$$

where $f(\beta, t)$ is the deterministic part which is a nonlinear function of the unknown vector of parameters $\beta \in R^k$, and other variables time-dependent included in $x(t)$. The component $\varepsilon(t)$ is a stochastic process representing the i.i.d. residual error. The usual assumptions consider $\varepsilon(t)$ as a white noise process. A further specification is based on the assumption of normality.

Firstly, a simple nonlinear least squares parameters estimation method (for

which it is sufficient the hypothesis of nullity of the mean of error components) can be implemented directly, see [31]. In a second phase, we examine the residuals of nonlinear regression.

If residuals do not support the hypothesis of a white noise process it is possible to consider the well-known transfer function models (ARMAX), proposed by [10] in order to approximate dynamic relationships between an input x_t and an output z_t , see for instance [17].

Therefore we estimate parameters β of Equation (4) following a non linear least squares procedure (e.g. Marquardt, Gauss–Newton or other criteria). At a second step, we use the estimated function $f(\hat{\beta}, t)$ instead of the regressor X_{t-b} , or a lagged multiplicity of regressors, in a ARMAX model. The solution is clearly sub-optimal if compared with an estimation procedure which takes into account jointly all the parameters. The lack of fit is however quite limited.

It is not a good choice that a global optimizer determine which are the most important components in model building because this fact may generate a local good fitting with no meaning.

In particular, in GBM modelling there is a prominent role of *natural diffusion* which provides a start–up of the process (innovative component) and iteratively initializes the imitative or logistic component of diffusion which assures the unimodal curvature before saturation.

A second level in the model construction is surely devoted to the deterministic identification of systematic perturbation $x(t)$. This is not a minor problem. It may strongly interact with the *natural diffusion* by shifting peaks, delaying or anticipating saturations. Non linear estimation with, say, 15 or 20 parameters

is not a simplistic matter and it requires theoretical knowledge of specific inference and good experience in software driving.

5 World Oil Depletion Models: interventions and price effects

In this Section we analyze the goodness-of-fit of the models introduced in Section 3 for data provided by Industriedatenbank from 1900 to 1986 included. The second part of the series, 1987 - 2002, is based on BP (British Petroleum) data in order to recover the new contribution of NGL (*Natural Gas Liquids*). We consider daily world oil production in thousands of barrels, from 1900 to 2002. The same source, BP, gives the series of barrel prices $D(t)$, with $t \in [1900 - 2002]$ in USA dollars (fixed base: 2002).

We have performed a wide number of transformations of the prices series $D(t)$ in order to explain systematic perturbation in the series $z'(t)$ of daily world oil production. Figure 1 may suggest many competing relationships.

We have considered various models for $x(t)$ following different economical reasoning. Main results are summarized in Table I where we report the performed models, with D' that is the function of prime difference of D , the R^2 and the residual sum of squares (SSE).

TABLE I ABOUT HERE

The performance is not completely satisfactory because experience about these nonlinear diffusion model teaches that an optimal configuration can be reached only when the R^2 is at least greater than 0.99995. A direct inspection of the graphical representation of both the observed data and each of the performed

model gives a strong evidence in order to reject them. Table 1 highlights that case a) has a global poor fitting. Cases b), c), d) and e) are characterized by a good performance in the first part of the series from 1900 to about 1970 and a very poor performance in the sequel with an oscillating structure of prediction that is incoherent with data. Finally, cases f) and g) present a very high perturbation in the central part of the series.

FIGURE 2 ABOUT HERE

These results suggest that prices series alone may not control completely such type of expressed demand. Moreover, the direct examination of data, see Figure 1, suggests a self-evident contradictory behaviour: sharp positive spikes in prices correspond to local increment in expressed demand. This is incoherent with standard economic theory of quantity-price relationship.

The second class of models is based on the hypothesis that the historical shocks emerged during natural evolution of oil production must be modeled separately and/or jointly with prices data. We have considered here various models for $x(t)$ following different historical and economical hypotheses. Main results are summarized in Table II that reports the performed models $x(t)$, the R^2 and the SSE , where $E_i = c_i \exp(b_i(t - a_i))$. Sometimes, the regressive implementation of the model is improved with an ARMAX model based upon one regressor or more lagged regressors depending upon the predicted values of the first regressive step.

TABLE II ABOUT HERE

Here we skip some statistical improvements of interpretation based upon linearized confidence interval even if some better work may be done considering

simultaneous exact confidence regions (see, for instance, [15], [7]) in order to examine intrinsic curvature aspects of solution locus. Weighting criteria depending upon heteroskedastic dispersion proportional to instantaneous effects, $\sigma(t) = \sigma z'(t)$, has been used to improve goodness-of-fit performance with different local regimes.

By analyzing Table II we see that model a), discussed for the first time in [14], is statistically well fitted (note that the R^2 reaches a good level). In this framework the world oil peak is estimated to occur in 2006 and a 90% depletion time is located around 2041 under uniform and regular interventions in the period following 2002. If we compare these new results with those obtained in [18] we see that a further positive exponential shock emerged around 1964. In this sense the strong intervention after 1979 may be thought of as a compensating element in order to correct the system to a standard regime. In particular, the coefficients c_i are similar but opposite in sign. The coefficients b_i are negative.

An instantaneous representation of observed and predicted values by models a), j) and k) is summarized in Figure 2.

FIGURE 3 ABOUT HERE

Models b) and c) are good improvements of residual autodependence structure. As we can see, the SSE decreases of a factor 10 but this type of modelling may interact with possible omitted systematic components of control function $x(t)$ as we will see later on. Model d) considers a simple combined effect of two shocks and the relative variation of prices $(\ln(D))' = D'/D$. The partial contribution of price relative variation is not significative. Models g) and h) highlight the effect of adding price series to previous two shocks model by

ratio $(1/D)'$ or both terms $(1/D)' + D'/D$. In these cases, squared partial correlation is about $\tilde{R}^2 = 0.057$ ($F \simeq 5.267$) and, respectively, $\tilde{R}^2 = 0.116$ ($F \simeq 5.368$), denoting a weak significance of prices after shock absorption of main perturbations. Improvements in model e) and f) follow the previous comments for similar situations. Model i) exhibits a good performance if compared with model g).

FIGURE 4 ABOUT HERE

GBM with 3 exponential shocks

The most interesting and surprising result is obviously represented by model j) in which we directly attempt to accommodate three shocks. If we compare model a) with model j) we find that partial multiple correlation coefficient is $\tilde{R}^2 = 0.56$, ($F \simeq 17.06$), so that the evidence of a third shock is very strong (compare Figure 3 with Figure 4). Let us summarize the main estimates in Table III.

TABLE III ABOUT HERE

We note a structural change of the memory of interventions. All coefficients b_i , $i = 1, 2, 3$ are positive so that the effects are persistent in time and interact significantly with the main stream of normal evolution controlled by standard Bass structure. The beginning times of the shocks are correctly positioned and this fact allows clear interpretations.

The starting point of new global economy effort begins with 1951 after the ruins of World War II and previous crisis such as that of 1929. The growth of economy is sustained by an exponential positive deviation of crude oil produc-

tion until 1974 (1973 for historical references). In 1973 there is the first long memory negative correction (Yom Kippur war and related embargo) followed by the second one in 1980 (1979, OPEC limitations and a decade of wars in Middle East region). The consequences of this new modelling are impressive. The peak time of maximum instantaneous production, 2007, is shifted of one year w.r. to the corresponding peak time based on a GBM with two exponential shocks. The effect of long memory perturbations is recognized in the identification of depletion times, $t_{0.90} = 2019$ and $t_{0.95} = 2023$, which appear particularly imminent.

If we look at these results from a statistical point of view, we deduce that Hubbert's approach based upon a *pure logistic equation* seems too much simplistic. Systematic perturbations may introduce a definitive modification in the production framework.

In order to understand these effects, see Figure 2 where heavy right tail belongs to a forecast based on a GBM with two exponential shocks with negative memory coefficients. The shorter forecasted curve is based upon a GBM with three exponential shocks and long memory effects sustained by positive coefficients.

In model k) we consider the marginal effect of prices added to a GBM based upon three shocks. The extension is very poor with a very low squared partial correlation, $\tilde{R}^2 = 0.01052$.

FIGURE 5 ABOUT HERE

The special position of the shock arising in 1951 is here highlighted, for the first time, with a simple model, GBM with three exponential shocks, that

extends the pioneering Hubbert's work. See for instance Figure 5 where the dots represent the daily oil production per year, the broken line depicts the proposed GBM model and the continuous line represents the evolution of the Bass process without interventions.

The subtle and shifty deviation originated in the post-war period beyond 1951 has generated an incremental consumption of oil not fully balanced by the feedbacks of 1970's. This deviation from the Hubbert model (exactly centered on the world oil data until 1950) is recovered by the GBM model with three shocks with an abrupt contraction of the diffusion process depletion component. The contraction is quite severe. The present model forecasts an URR of 1524 Gbo, a peak positioned in 2007 with a production of 76.34 mb/d, a depletion of 90% URR in 2019 with a production of 55.33 mb/d and a 95% saturation during 2023 with a production of 36.13 mb/d.

Note that the peak-date, by 2007, and the estimated production during 2020 by about 55 mb/d are exactly equivalent to those referred in [2] by the means of WOCAP simulations.

6 Final remarks

The explicative or anticipative role of annual average prices of crude oil are generally weakly significative. Both in model h) and model k) we note the coherent sign (negative) that depicts a classical negative relation quantity-price according to the standard economic theory. Nevertheless, the analysis based on model j), i.e., GBM with three shocks, excludes a central role of prices of crude oil in determining the decisions of its use. Crude oil is the

primary energy source of this short historic period (one century and half) and its life time is not yet so long.

The structural change of sign of coefficients b_i , $i = 1, 2, 3$, highlights the central role of interventions with respect to normal evolution of the production process of crude oil. The acceleration of the learning of the social and economic system is a stable modification of standard Bass evolution. The modification is permanent and non absorbable as well as in a usual decay process of the memory of an event (negative coefficients).

The actual evolution of oil production, after two negative shocks, is still under the influence of the first shock (1951) started with the first post-war period. This push added to the natural evolution of diffusion gives rise to a contraction of the right tail of distribution over time compared with GBM estimated models with one or two exponential shocks. The peak time is delayed, 2007, but saturation is strongly anticipated, $t_{0.90} = 2019$ and $t_{0.95} = 2023$.

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Table 1

World oil depletion: GBM models with pure prices control.

n.	$x(t)$ control model	R^2	SSE
a):	$1 + cD - a$	0.999624	2.25264E10
b):	$1 + aD'/D$	0.999822	3.84188E09
c):	$1 + aD'$	0.999813	1.11859E10
d):	$1 + cD - a + bD'/D$	0.999892	6.49115E09
e):	$1 + aD' + bD'/D$	0.999820	1.07567E10
f):	aD	0.978043	1.31507E12
g):	a/D	0.988635	6.80707E11

Table 2

World oil depletion: GBM models with shocks and prices control.

n.	name	$x(t)$ control model	R^2	SSE
a):	gbm2e	$1 + E_1 + E_2$	0.999988	7.24271E08
b):	gbm2e, armax(4,4)	$1 + E_1 + E_2, \text{lag} = 0$		6.25567E07
c):	gbm2e, armax(2,0)	$1 + E_1 + E_2, \text{lag} = 2$		6.79209E07
d):	gbm2elnD	$1 + E_1 + E_2 + a(\ln(D))'$	0.999988	7.06147E08
e):	gbm2elnD, armax(4,4)	$1 + E_1 + E_2 + a(\ln(D))'$		7.48614E07
f):	gbm2e+f(D), armax(4,0)	$(1 + E_1 + E_2)' + D' + (\ln(D))'$ inst. data		6.77601E07
g):	gbm2eDra	$1 + E_1 + E_2 + a(1/D)'$	0.999989	6.83253E08
h):	gbm2eDralnD	$1 + E_1 + E_2 + a(1/D)' + b(\ln(D))'$	0.999989	6.39745E08
i):	gbm2eDrarun	$1 + E_1 + E_2 + a(1/D)$	0.999989	6.62275E08
j):	gbm3e	$1 + E_1 + E_2 + E_3$	0.999994708	3.16947E08
k):	gbm3eDralnD	$1 + E_1 + E_2 + E_3 + a(1/D)' + b(\ln(D))'$	0.999994764	3.13612E08

Table 3

World oil depletion: GBM estimates with three exponential shocks.

$m = 4174561$	$p = 0.00010439$	$q = 0.063497$
$c_1 = -0.3021860$	$b_1 = 0.05674$	$a_1 = 80.50$
$c_2 = 0.0717753$	$b_2 = 0.07187$	$a_2 = 51.07$
$c_3 = -0.2272032$	$b_3 = 0.07098$	$a_3 = 74.60$

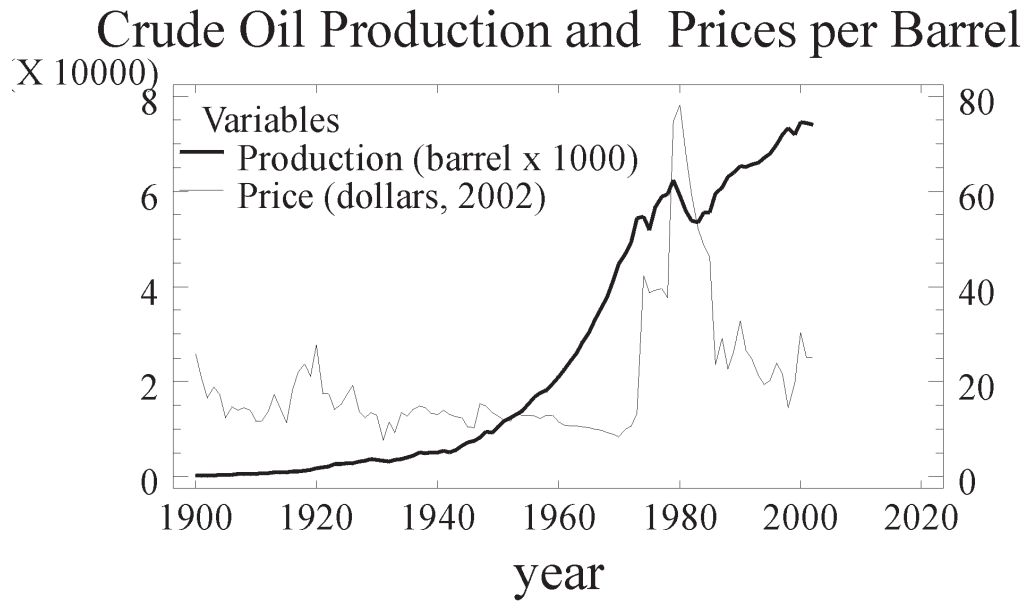


Fig. 1. World oil daily production (thousands of barrels) and prices per barrel (dollars, 2002); Source: Industriedatenbank and British Petroleum, BP.

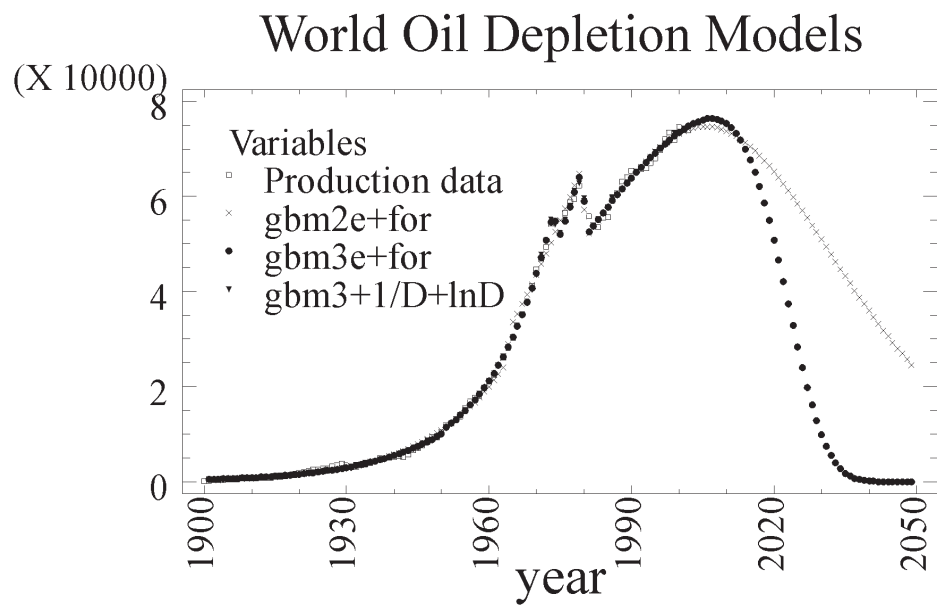


Fig. 2. World Oil Depletion Models: GBM under two or three shocks and price effects.

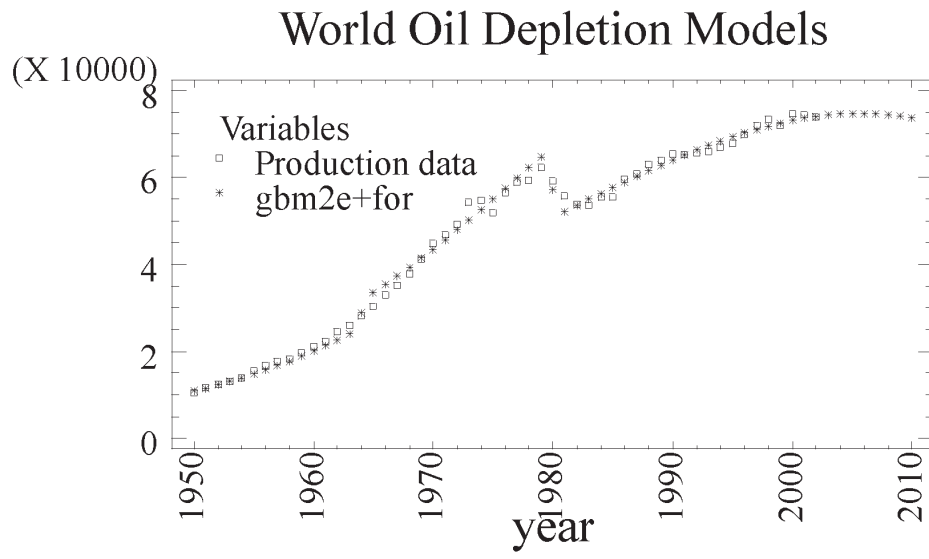


Fig. 3. World Oil Depletion: a zoom on GBM after 1950 with two exponential shocks.

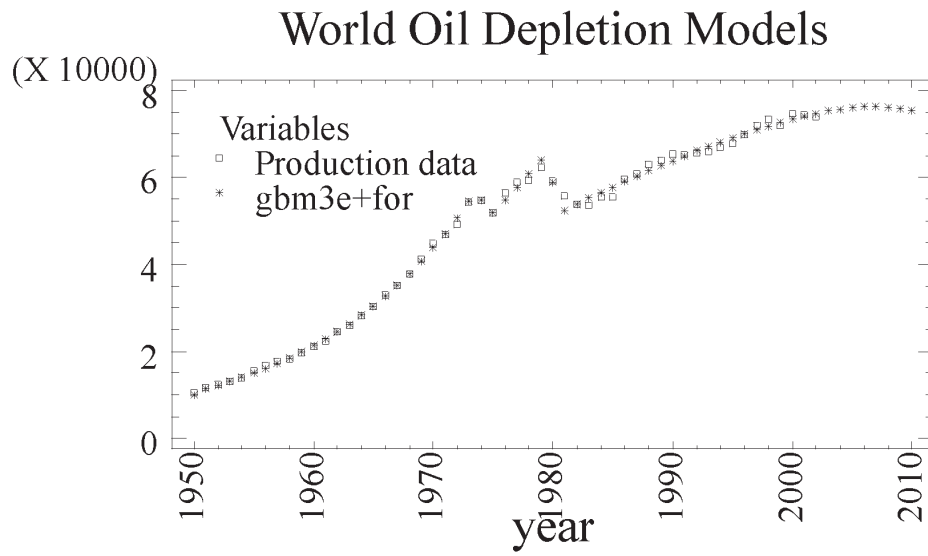


Fig. 4. World Oil Depletion: a zoom on GBM after 1950 with three exponential shocks.

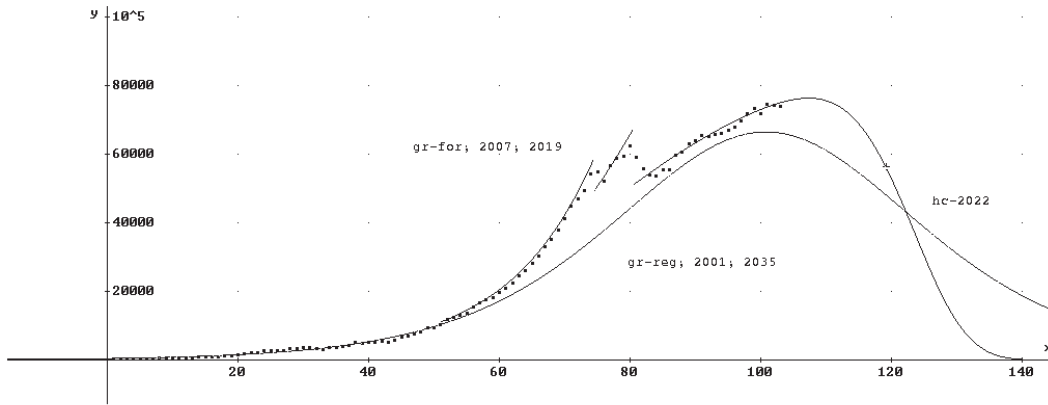


Fig. 5. World Oil Depletion: GBM with three shocks vs Hubbert–Bass model.