Market potential dynamics in innovation diffusion: modelling the synergy between two driving forces☆

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Abstract

The presence of a slowdown in new product life cycles has recently received notable attention from many innovation diffusion scholars, who have tried to explain and model it on a dual-market hypothesis (early market-main market). In this paper we propose an alternative explanation for the slowdown pattern, a dual-effect hypothesis, based on a recent co-evolutionary model, where diffusion results from the synergy between two driving forces: communication and adoption. An analysis of the synergistic interaction between communication and adoption, based on the likelihood ratio order or on a weak stochastic order, can inform us of which of the two had a driving role in early diffusion. We test the model on the sales data of two pharmaceutical drugs presenting a slowdown in their life cycle and observe that this is identified almost perfectly by the model in both cases. Contrary to the general expectation, according to which communication should precede adoption, our findings show that adoptions may be the main driver in early life cycle; this may be related to the drug's specific nature.

Keywords:
Co-evolutionary diffusion process, Dual-effect market, Communication network, Slowdown, Saddle, New drugs, Likelihood ratio order

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1. Introduction

The literature on innovation diffusion and new product life cycle has highlighted that in many situations the diffusion process is not as smooth as one would expect, but rather presents a perturbed pattern. In particular, it has been observed that the growth phase of the process is often characterised by the presence of a *slowdown*. The slowdown phenomenon -also known, with minor differences, as chasm, saddle or dip- indicates the situation in which, after a rapid takeoff, a product's sales reach an initial peak followed by a decline -whose length and depth may vary- and eventually by a resumption that may exceed the initial peak. While in the past there was no consensus on the concrete existence of such a phenomenon, a recent and increasing stream of literature has empirically verified that this regularity occurs in many product categories. However, the slowdown phenomenon is still posing challenges to innovation diffusion scholars, since it has been neither explained nor modelled in a unique way.

Some lines of research have followed the idea that the market for new products needs to be divided into two major segments, usually termed "visionaries and pragmatists" (see [1]), "early market and main market" (see [2], [3], [4], [5]), "influentials and imitators" (see [6]).

In particular Moore, building on the well-known categorisation of adopters proposed by [7], suggested that the market for innovations is initially represented just by early adopters and that the main market develops in a second stage of diffusion. Early and main markets are different in their attitudes and expectations towards novelties, and this difference may result in a precise separation between the two, implying a different treatment in terms of marketing strategies (see [8]). Such a separation has been theorized as a possible explanation for the slowdown pattern. Grounding on Moore's intuition, Goldenberg et al. (see [2]) have suggested that the existence of a saddle may be seen as a dual-market symptom. Their analysis has been based, first, on two exploratory studies on artificial markets realized with Cellular Automata models in order to verify the frequency of the saddle phenomenon in simulated situations, and then on an aggregate model to tie the dual-market explanation to saddle phenomena emerging in real situations.

In the spirit of the work by [2], Muller and Yogev (see [3]) have developed a dual-market diffusion model, in which the dynamics of the early market are expressed in Equation (1),

$$\frac{dI(t)}{dt} = \left(p_i + q_i \frac{I(t)}{N_i}\right)(N_i - I(t)). \tag{1}$$

As one may observe the early market's cumulative adoptions, $I(t)$, are described through a simple Bass model, (BM) [9], where parameters p_i and q_i have the usual meaning and N_i is the market potential of the early market. Instead, cumulative adoptions of the main market, $M(t)$, present a more complex structure,

$$\frac{dM(t)}{dt} = \left(p_m + q_m \frac{M(t)}{N_i + N_m} + q_{im} \frac{I(t)}{N_i + N_m}\right)(N_m - M(t)). \tag{2}$$

Equation (2) proposes a bipartition of the word-of-mouth effect, which would be partly due to communication among the main market's individuals, q_m, and partly to cross-market communications between the early and the main markets, q_{im}.

Karmeshu and Goswami (see [4]) have introduced a different methodology in order to take into account the heterogeneity of agents in a standard *normalised* Bass model, (BM),

$$\frac{dX(t)}{dt} = \alpha(1 - X) + \beta X(1 - X), \qquad X(0) = X_0 \tag{3}$$

by modifying its basic structure via a general assumption about the stochastic nature of α and β parameters. The solution process corresponding to model (3) is the usual one based conditionally on α, β, and X_0. The authors study particular moments in the previous process describing a general joint mixing distribution $\phi(\alpha, \beta)$ by means of so-called 'two-point-distribution' (TPD) formalism. This allows for an approximate representation through six parameters, μ_i, σ_i, ν_i, $(i = \alpha, \beta)$, i.e., local means, standard deviations, and skewness. This is an innovative approach which allows a formal definition of the dynamic mean value $\mathcal{M}(t)$ of the cumulative adoption process as a linear combination of four Bass standard cumulative distributions. Studying the variation of $\frac{d\mathcal{M}(t)}{dt}$ with reference to time t for various choices of parameters, it is possible to describe unimodal and bimodal life cycles where the latter is obtained for increasing values of standard deviations σ_α and σ_β. This decomposition allows a flexible description of diffusion evolution, but not a clear and interpretable origin of the components dominating over time.

Following a different path, Van den Bulte and Joshi (see [6]) have recently dealt with the existence of a dual market and the slowdown in diffusion. The authors have developed a two-segment mixture model to account for the presence of two distinct subpopulations, namely influentials and imitators, whose adoption behaviour is captured by the following hazard functions,

$$h_1(t) = p_1 + q_1 F_1(t), \tag{4}$$

$$h_2(t) = p_2 + q_2[wF_1(t) + (1-w)F_2(t)]. \tag{5}$$

Consistently with the influentials–imitators hypothesis, Equations (4) and (5) show an asymmetry; in fact, type 1 may influence type 2, but the reverse cannot occur. The overall adoption process is the weighted sum of the adoption of the two segments, under the assumption that these may not have the same importance, i.e.,

$$F_m(t) = \vartheta F_1(t) + (1-\vartheta)F_2(t), \tag{6}$$

where $F_1(t)$ and $F_2(t)$ are probability distribution functions. Similarly, the weighted sum of the corresponding densities yields

$$f_m(t) = \vartheta f_1(t) + (1-\vartheta)f_2(t). \tag{7}$$

The so-called Asymmetric Influence Model (AIM) by Van den Bulte and Joshi is defined by calculating closed-form solutions of $F_1(t)$ and $F_2(t)$. The solution of $F_1(t)$ is that of the standard Bass model, while $F_2(t)$ presents a much more complex structure, referable to a Riccati equation. Though we do not report the details of such a solution, the impressive mathematical effort of the overall construction is noteworthy indeed. Abandoning the closed-form solution of F_2, the authors have proposed a numerical solution for Equation (8),

$$\frac{dX(t)}{dt} = M[\vartheta f_1(t) + (1-\vartheta)f_2(t)] + \varepsilon(t). \tag{8}$$

Van den Bulte and Joshi's model definitely proposes a mixture of two subpopulations of adopters as a possible explanation for the chasm (or dip) exhibited by several diffusion processes: specifically such a pattern appears when considering Equation (7), i.e., the weighted sum of two densities.

In this paper we propose an alternative explanation for the slowdown pattern that we define as a *dual-effect hypothesis*, based on a co-evolutionary innovation diffusion model by [10], where diffusion results from the synergy between two forces, communication and adoption. In particular, communication is necessary to create the market potential. By studying the properties of this model in depth, we highlight that a slowdown emerges from the co-evolution of communication and adoption, when these two components are significantly separate over time. This analysis is also very useful to show that communication does not necessarily precede adoption, as one would expect, but in some cases the reverse may occur. To illustrate these opposite situations, we present two paradigmatic cases referring to the life cycle of new drugs, providing a plausible explanation for the different behaviour observed.

The paper is organised as follows. In Section 2 we summarise the basic properties of the co-evolutionary model by [10] with a particular specification of a dynamic market potential. A natural decomposition of the model density allows a direct interpretation of the evolutionary drivers due to communication and adoption forces. The *Likelihood ratio order* explains the different time position of these forces. A further *weak ordering*, based on simple location indexes, is proposed and compared with the likelihood ratio order or the *usual stochastic order*. In Section 3 we consider two different pharmaceutical drug diffusions in the Italian market that exhibit slowdowns and saddle effects well-recognised by the previous model. In Section 4 we analyse the time positioning of communication and adoption, and propose a possible interpretation for the obtained results. Final comments and discussion are presented in Section 5. Appendix A examines four other applications in order to confirm the proposed results.

2. Co-evolution of Market Potential and Diffusion of an Innovation

The model introduced in [10] is based on a special Cellular Automata (CA) description whose aggregate mean-field approximation, in continuous time, yields

$$y'(t) = m(t)\left\{-r_s\frac{y(t)}{m(t)} + \left(p_s + q_s\frac{y(t)}{m(t)}\right)\left(1-\frac{y(t)}{m(t)}\right)\right\}x(t) + y(t)\frac{m'(t)}{m(t)}, \tag{9}$$

where $y'(t)$ represents instantaneous adoptions at time t, $y(t)$ denotes the corresponding cumulative adoptions, p_s and q_s are the usual Bass-like parameters depicting innovation (external) and imitation (internal) effects, r_s

accounts for a possible decay effect due to unretained adoptions (subscript s refers to adoptions or sales). Function $m(t)$ is the dynamic market potential, and function $x(t)$ (see [11]) represents an intervention tool (environmental or strategic perturbations), which modifies the adoption process by expanding, $x(t) > 1$, or reducing, $x(t) < 1$, the residual market, $m(t) - y(t)$.

Equation (9) defines a nested co-evolutionary model as a special non-autonomous Riccati equation. Its closed form solution (see [10]), is

$$y(t) = m(t) \frac{1 - e^{-D_s \int_0^t x(\tau)d\tau}}{\frac{1}{{}_s r_2} - \frac{1}{{}_s r_1} e^{-D_s \int_0^t x(\tau)d\tau}}, \quad D_s = \sqrt{(q_s - p_s - r_s)^2 + 4q_s p_s} > 0, \tag{10}$$

where ${}_s r_i = (-(q_s - p_s - r_s) \pm D_s)/(-2q_s),\ i = 1, 2$, with ${}_s r_2 > {}_s r_1$. The second factor in Equation (10) describes *adoption dynamics* under the modulation of the market potential factor, $m(t)$. In this model particular attention is devoted to a general definition of the market potential through a non-negative function $m(t) \geq 0$, which may be modelled in different ways. In [10] the following structure, based on a Cellular Automata (CA) representation, is proposed,

$$m(t) = K \sqrt{\frac{1 - e^{-(p_c + q_c)t}}{1 + \frac{q_c}{p_c} e^{-(p_c + q_c)t}}}. \tag{11}$$

where p_c and q_c respectively denote the external and internal components of the *communication process*, while K is the asymptotic market potential. The final model, in its reduced form (with $r_s = 0$ and $x(t) = 1$), as proposed in [10], is

$$y(t) = K \sqrt{\frac{1 - e^{-(p_c + q_c)t}}{1 + \frac{q_c}{p_c} e^{-(p_c + q_c)t}}} \frac{1 - e^{-(p_s + q_s)t}}{1 + \frac{q_s}{p_s} e^{-(p_s + q_s)t}}. \tag{12}$$

The flexible character of model (12) is very interesting for its interpretability (unimodality, slowdown, and saddle effects). Figure 1 shows three different configurations of model (12) for the instantaneous process, $y'(t)$. In particular, cases A and B have two peaks, with a good slowdown and a saddle. In case B, the higher value of parameter p_c (0.045) increases earlier adoptions. Case C, characterised by higher adoption parameters, generates a uni-modal behaviour without slowdown and with a weaker right tail.
A simple reparameterisation of Equation (12), i.e., $a = p_s + q_s$, $b = q_s/p_s$,

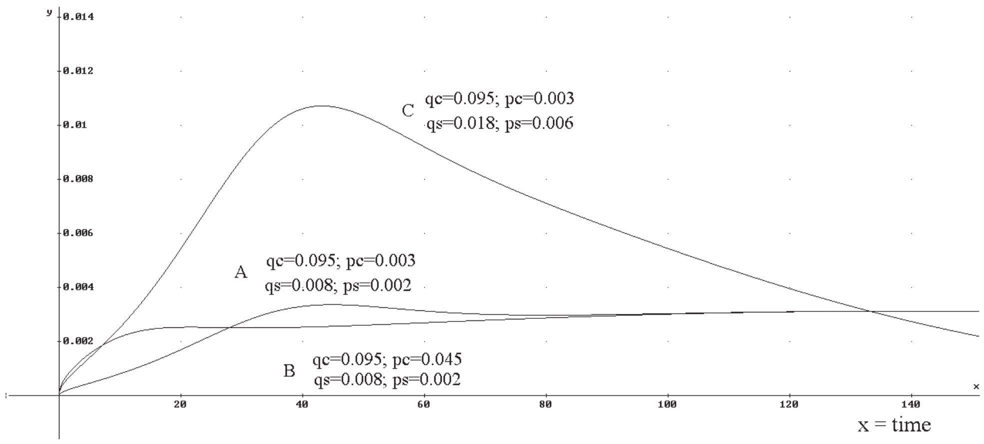

Figure 1: Three different co-evolutionary instantaneous adoption processes. Case B depicts earlier adoptions than case A in the first part of the cycle due to a higher p_c parameter. Both A and B present slowdown and saddle effects. Case C shows a uni-modal distribution over time with a short right tail.

$c = p_c + q_c$ and $d = q_c/p_c$, yields

$$y(t) = K \cdot \sqrt{\frac{1-e^{-ct}}{1+de^{-ct}}}\ \frac{1-e^{-at}}{1+be^{-at}} = K \cdot \sqrt{F(t)}\, G(t), \tag{13}$$

under the usual assumptions on diffusion parameters, $0 < p_s < q_s$ and $0 < p_c < q_c$. Function $K(t) = \sqrt{F(t)}\, G(t)$, different from scale parameter K, is a probability distribution, product of two d.f., $\sqrt{F(t)}$ and $G(t)$, describing communication and adoption processes respectively.

Let us denote its density $k(t) = dK(t)/dt$ with a reduced notation illustrating a *dual-effect decomposition*, i.e.,

$$k(t) = \frac{1}{2}F(t)^{-1/2}G(t)f(t) + F(t)^{1/2}g(t) = k_1(t) + k_2(t), \qquad t > 0, \tag{14}$$

where $f(t) = dF(t)/dt$ and $g(t) = dG(t)/dt$ are the densities of communication and adoption processes, respectively. Equation (14) states that density $k(t)$ is the sum of two non-negative components, $k_1(t)$ and $k_2(t)$ which, in turn, are monotone transformations of densities $f(t)$ and $g(t)$, respectively. Thus we associate $k_1(t)$ with $f(t)$ and take it as a modified communication component. Similarly, we associate $k_2(t)$ with $g(t)$ as a modified adoption component. We may consider a normalisation of non-negative functions $k_1(t)$ and $k_2(t)$ deriving two corresponding densities $\tilde{k}_i(t) = k_i(t)/K_i$ with

$K_i = \int_0^\infty k_i(t)dt,\ i = 1, 2$. Our aim is to study the temporal allocation of $\tilde{k}_1(t)$ and $\tilde{k}_2(t)$, in order to understand which of the two "comes first".

Suppose that a random variable X is associated with $\tilde{k}_1(t)$ and a random variable Y with $\tilde{k}_2(t)$.

Definition. We say that Y is *larger* than X in *likelihood ratio order*, $X \leq_{lr} Y$, if X and Y have densities such that, for all $s \leq t$,

$$\tilde{k}_1(t) \cdot \tilde{k}_2(s) \leq \tilde{k}_1(s) \cdot \tilde{k}_2(t). \tag{15}$$

Notice that the inequality based on $\tilde{k}_i(t),\ i = 1, 2$ densities does not depend on the quantities K_1 or K_2 or their ratio, so that we can directly compare $k_i(t),\ i = 1, 2$. Equation (15) states that $\tilde{k}_2(t)/\tilde{k}_1(t)$ or $k_2(t)/k_1(t)$ is increasing, avoiding the special cases with vanishing denominators.

The direct use of the *likelihood ratio criterion* gives a strong result for the temporal ordering of $k_1(t)$ and $k_2(t)$, in order to recognise the dominance between communication and adoption. Nevertheless, this ratio requires plotting features to be implemented. We propose, as an alternative procedure, some location indexes for detecting such a dominance, in a *weak order* sense. A random variable X is smaller than a random variable Y under the *usual stochastic order*, $X \leq_{st} Y$, if the corresponding distribution functions satisfy, for all t, inequality $F_X(t) \geq F_Y(t)$. It is well-known that the *likelihood ratio order* is stronger than the *usual stochastic order*, i.e., if $X \leq_{lr} Y$ then $X \leq_{st} Y$ (see, e.g., theorem 1.4.4 in [13]).

In order to realise a temporal positioning of $f(t)$ and $g(t)$, we may define two random variables in time domain, T_F and T_G, with distribution functions $F(t)$ and $G(t)$, mentioned in Equation 13. Observe that the *usual stochastic order* between T_F and T_G, e.g., $\leq_{st}$, implies an analogous (total) *weak order* of the corresponding mean values, $E(T_F) \leq E(T_G)$, associated with $f(t)$ and $g(t)$, respectively (see, e.g., theorem 1.2.9 in [13]). We may use different location indexes related to the random variables T_F and T_G, the mode, t^+, the median, $t_{0.5}$, and the mean value, $\bar{t}$, i.e.,

$$\begin{aligned}
t^+_{com} &= \frac{\ln d}{c}; \qquad {}_F t_{0.5} = \frac{1}{c}\ln(2+d); \qquad E(T_F) = \bar{t}_F = \frac{1}{q_c}\ln(1+d), \\
t^+_{ado} &= \frac{\ln b}{a}; \qquad {}_G t_{0.5} = \frac{1}{a}\ln(2+b); \qquad E(T_G) = \bar{t}_G = \frac{1}{q_s}\ln(1+b).
\end{aligned} \tag{16}$$

For $0 < p_c < q_c$ and $0 < p_s < q_s$ the location indexes within T_F and T_G -mode, median and mean value- are increasing values, $t^+ < t_{0.5} < \bar{t}$. Moreover, we can prove that the *likelihood ratio order* or the *usual stochastic order* imply an equivalent *weak order* in terms of medians: $T_F \leq_{st} T_G \Rightarrow {}_Ft_{0.5} \leq {}_Gt_{0.5}$.

The reverse version of the above-mentioned theorem 1.2.9, or an equivalent form for medians, is not true in general. Nevertheless, the total *weak order* based on mean values or medians or modes (16) may be a strong symptom of the existence of an analogous *usual stochastic order* between $k_1(t)$ and $k_2(t)$. Therefore, we can obtain some information on the relative time position of $k_1(t)$ and $k_2(t)$ by diagnosing it via pairs of corresponding location indexes, referred to $f(t)$ and $g(t)$. In particular, observing that $t^+_{com} < t^+_{ado}$, ${}_Ft_{0.5} < {}_Gt_{0.5}$, and $\bar{t}_F < \bar{t}_G$, we may conclude that we are facing the most common situation where the communication process is the main driver of a possible take-off in diffusion, so that we have T_F *smaller* than T_G, following a location *weak order* denoted by the symbol $<<$, $k_1(t) << k_2(t)$. The actual order may be a stronger one, i.e., $\leq_{lr}$ or $\leq_{st}$.

Vice versa, for $t^+_{com} > t^+_{ado}$, ${}_Ft_{0.5} > {}_Gt_{0.5}$, and $\bar{t}_F > \bar{t}_G$ there is a reverse order in time domain, i.e., in this case the adoption component is the main driver, and communication gives rise to a maintenance effect.

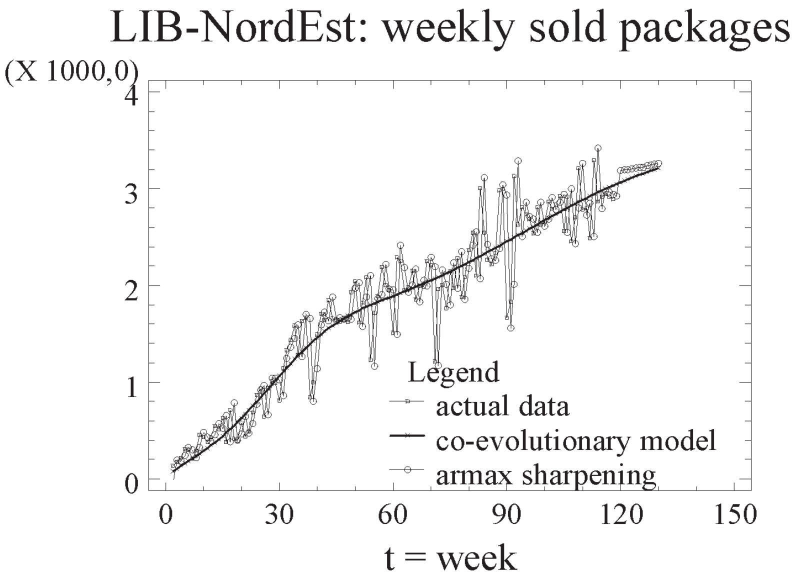

Figure 2: LIB-NordEst, Italy: Co-evolutionary non-cumulative model with no exit rule, ARMAX sharpening, and actual weekly “LIB” packages sold.

Table 1: Pharmaceutical drugs' diffusion. Parameter estimates of co-evolutionary models for "LIB-NordEst" and "LYR-Italy". ( ): marginal linearised asymptotic 95% confidence limits

"LIB-NordEst"						
K	q_c	p_c	q_s	p_s	R_1^2	*indexes*
647061	0.08114	0.00385	0.01853	0.00100	0.999939	$DW = 0.264$
(533821)	(0.07483)	(0.00345)	(0.01696)	(0.00088)	SSE :	$P^2 = 0.956920$
(760300)	(0.08746)	(0.00427)	(0.02010)	(0.00112)	[2.88209E7]	$F = 1255$
"LYR-Italy"						
K	q_c	p_c	q_s	p_s	R_1^2	*indexes*
5116020	0.05322	0.00090	0.09451	0.03408	0.999908	$DW = 0.172$
(5066630)	(0.05207)	(0.00087)	(0.07987)	(0.03056)	SSE :	$P^2 = 0.865300$
(5165410)	(0.05438)	(0.00093)	(0.10914)	(0.03759)	[1.77303E10]	$F = 296$

Table 2: LIB-NordEst, Italy: Co-evolutionary cumulative model with no exit rule and ARMAX(1,0,1) sharpening. ( ): t-statistic; [ ]: p-values

$AR(1)$	MA(1)	$libNEcoevCM$	$mean$	SSE	indexes
0.84483	-0.20266	1.0101	1051.21	7.027450E6	$R_2^2 = 0.999985$
(14.75)	(-1.99748)	(375.787)	(3.7364)	$\{d.f.114\}$	$P^2 = 0.756168$
[0.000000]	[0.048155]	[0.000000]	[0.000293]		$F \simeq 114$

3. The Diffusion of Two New Drugs in Italy

In this section we present two applications of the co-evolutionary model by [10] to the diffusion of new pharmaceutical drugs, introduced in the Italian market in 2005, in order to detect the slowdown and the associated ordering of the dual-effect components.

The weekly data, provided by IMS Health, cover the period August 2005 through July 2007, with a spatial disaggregation by areas, and refer to the number of sold packages.

We examine these two different configurations, "LIB-NordEst", and "LYR-Italy" where the first part of the code refers to specific drugs, "LIB" and "LYR". In Appendix A we examine applications referring to other drugs, in order to confirm the main results presented in this Section.

Based on the active principle of *Barnidipine*, "LIB" is a new calcium-antagonist introduced in Italy in April 2005 for the treatment of mild-to-moderate hypertension. Barnidipine is the 12th calcium-antagonist put into commerce, and has been proven to be essentially equivalent to other, less recent calcium-antagonists.

Based on the active principle of *Pregabalin*, "LYR" was initially approved for treating epilepsy (as adjunctive therapy), neuropathic pain, and post-herpetic neuralgia pain, that is, to treat pain caused by nerve damage due to diabetes and herpes zoster infection.

We apply model (12) in its reduced form, with an additive noise $\varepsilon(t)$, with t interpreted in a discrete time domain, namely,

$$w(t) = K\sqrt{\frac{1-e^{-(p_c+q_c)t}}{1+\frac{q_c}{p_c}e^{-(p_c+q_c)t}}\,\frac{1-e^{-(p_s+q_s)t}}{1+\frac{q_s}{p_s}e^{-(p_s+q_s)t}}} + \varepsilon(t), \qquad t = 1, 2, \cdots . \quad (17)$$

The statistical implementation of model (17) may require alternative error structures. In a nonlinear regressive approach, we consider a particular model for observations, $w(t) = y(t)+\varepsilon(t)$, with an i.i.d. residual $\varepsilon(t)$. A more refined approach is based on ARMAX representation, $\Phi(B)(w(t)-\hat{y}(t)) = \vartheta(B)a(t)$, with $a(t)$ a white noise, where $\hat{y}(t)$ is a standard nonlinear regressive estimation as a first step, acting as "covariate" parallel to the autocorrelated residuals (see, for instance, [10] and [15]). Discrete parameters in ARMAX models are determined on the basis of the usual criteria: parsimony and residual whiteness tests. In Table 1 we summarise the estimation results for the proposed cases under a standard nonlinear least squares approach (Levemberg-Marquardt; see, for instance, [12]) in the co-evolutionary model as expressed in Equation (17).

In order to test the global significance of model (17), as compared with the standard BM characterised by a R_2^2 determination index, we can compute a nonparametric squared multiple partial correlation coefficient, $P^2 = (R_1^2 - R_2^2)/(1-R_2^2)$, normalised in the interval $[0, 1]$, and the corresponding F-ratio, $F = P^2(N-k)/[(1-P^2)s]$, where N is the number of observations, k the parameter cardinality of the co-evolutionary model, and $k-s$ the parameter cardinality of the nested model, BM (a common critical value for the F-ratio is 4). The same approach, based on P^2 and F-ratio, will be followed to compare the ARMAX extension to model (17).

As we may notice from the values of the determination index, R_1^2, the models present very high levels of global fitting and all the involved parameters are significant.

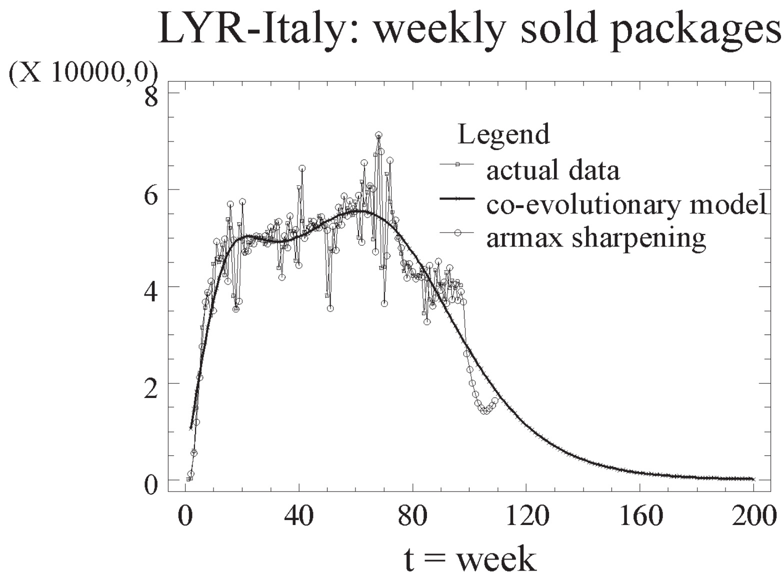

Figure 3: "LYR-Italy", Italy: Co-evolutionary non-cumulative model with no exit rule, ARMAX sharpening, and actual weekly "LYR" packages sold.

With reference to "LIB-NordEst" we observe a very good behaviour of the model with a noticeable NLS fitting, $R_1^2 = 0.999939$, confirmed by the squared partial correlation coefficient $P^2 = 0.956920$, and the corresponding ratio $F = 1255$, calculated with respect to the baseline BM. A low level of the Durbin-Watson statistic, $DW = 0.264$, denotes a high level of autocorrelation of residuals. With an ARMAX procedure we obtain a significant improvement, as one may see by the squared partial correlation, $P^2 = 0.76$, and F-ratio, $F \simeq 114$. Detailed results are reported in Table 2. Figure 2 highlights a weak but visible slowdown effect.

"LYR-Italy" presents a mature life cycle with some evidence of market contraction. The NLS fitting is excellent, $R_1^2 = 0.99991$, confirmed by the squared partial correlation coefficient $P^2 = 0.8653$ and the corresponding ratio $F \simeq 296$, calculated with respect to the baseline BM. The subsequent ARMAX(2,0,2) sharpening is quite effective, $P^2 = 0.8952$ and $F \simeq 188$ (see Table 3). The presence of a bimodal behaviour is well-recognised by the co-evolutionary model, with an evident saddle effect (see Figure 3).

A further improvement on the residual autocorrelated component, here omitted for simplicity, may be obtained through a SARMAX model, in order to absorb the small time lag between "actual data" and "ARMAX sharpening" as shown in Figures 2 and 3.

Table 3: "LYR-Italy", Italy: Co-evolutionary cumulative model with no exit rule and ARMAX(2,0,2) sharpening. ( ): t-statistic; [ ]: p-values

$AR(1)$	$AR(2)$	$MA(1)$	$MA(2)$	$lyrcevCM$	$mean$	SSE	indexes
1.8721	-0.9392	0.7180	0.2906	1.00071	-1384.72	1.8585021E9	$R_2^2 = 0.9999904$
(40.19)	(-20.343)	(8.6091)	(3.3500)	(2205.34)	(-1.1176)	$\{d.f.\ 91\}$	$P^2 = 0.8951793$
[0.0000]	[0.0000]	[0.0000]	[0.0012]	[0.0000]	[0.2667]		$F \simeq 188$

Previous results highlight different dynamics in the diffusion of these new pharmaceutical drugs. In both cases we report the presence of a slowdown effect in diffusion. The next Section will show that these effects may be explained through an interaction between communication (variable market potential) and corresponding adoption.

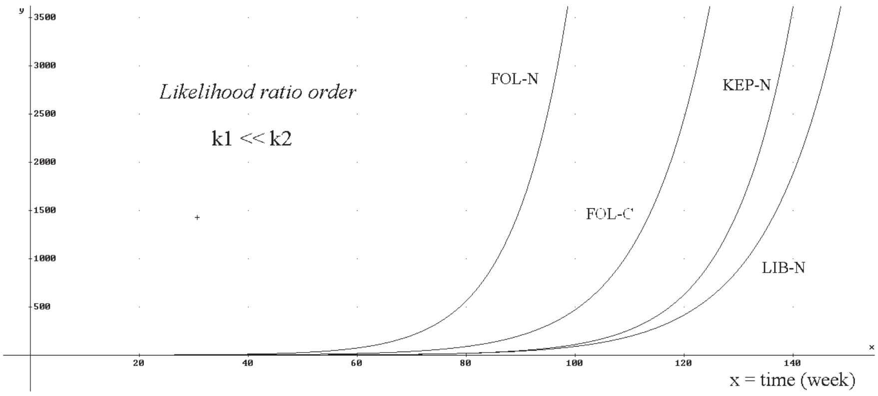

Figure 4: Likelihood Ratio Order between $k_2(t)$ and $k_1(t)$ for pharmaceutical drugs in Italy: "FOL-NordEst", "FOL-Centro", "LIB-NordEst" and "KEP-NordEst". The increasing ratio denotes a (first order) stochastic dominance of $k_2(t)$ or, equivalently, a driving role of communication, $k_1(t)$. Data source: IMS-Health, Italy. Normalised weekly packages sold; period: 8/2005 – 7/2007.

4. Pharmaceutical drugs in Italy: driving forces effects

In this Section we examine the pharmaceutical drugs reported in Section 3

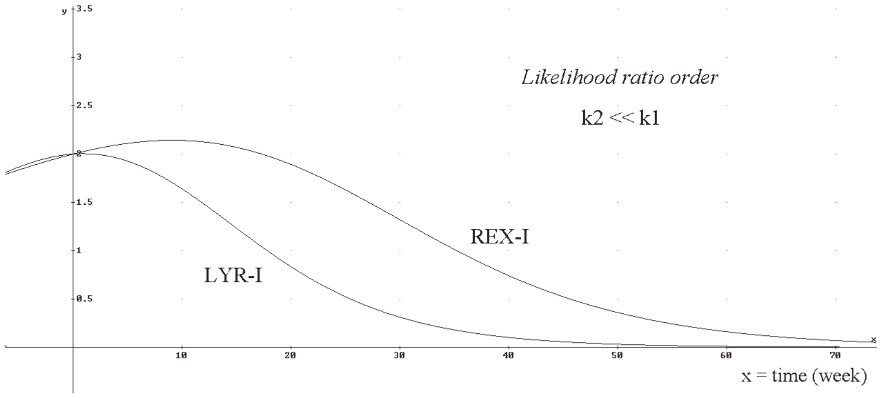

Figure 5: Likelihood Ratio Order between $k_2(t)$ and $k_1(t)$ for pharmaceutical drugs in Italy: "REX-Italy" and "LYR-Italy". The decreasing ratio denotes a (first order) stochastic dominance of $k_1(t)$ or, equivalently, a driving role of adoption, $k_2(t)$. Data source: IMS-Health, Italy. Normalised weekly packages sold; period: 8/2005 – 7/2007.

in order to detect and interpret the relative positioning of the two synergistic forces, communication and adoption.

As a direct control, we can compute the likelihood ratios $\tilde{k}_2(t)/\tilde{k}_1(t)$ or $k_2(t)/k_1(t)$ pertaining to the diffusions of "LIB-NordEst" and "LYR-Italy".

We report the results of the likelihood ratios in two separate plots. Figure 4 highlights a practically increasing behaviour of "LIB-NordEst", among other drugs analysed in Appendix A. This allows a simple interpretation: the effect associated with $k_1(t)$, i.e., communication, had a driving role in the evolution of this drug.

Vice versa, Figure 5 depicts the opposite diffusion structure of "LYR-Italy" with adoption forces in the driving role.

In Table 4 we summarise the basic estimates, the modal time values, t^+_{com}, t^+_{ado}, the median time values, ${}_Ft_{0.5}$, ${}_Gt_{0.5}$, and the mean values, $\bar{t}_F$, $\bar{t}_G$, referred to communication and adoption, respectively.

Observing Figure 6, we see that in the case of "LIB-NordEst" the weak order based on location indexes (Table 4) confirms that communication has a driving role, preceding and pulling adoptions. By contrast, in the case of "LYR-Italy", depicted in Figure 7, we observe an explicit inversion, so that the adoption component dominates the first part of the diffusion. This difference in behaviour may be related to the nature of the drugs considered.

"LYR" was originally developed for neuropathic pain, a symptom common to various pathologies that are extremely difficult to understand and to

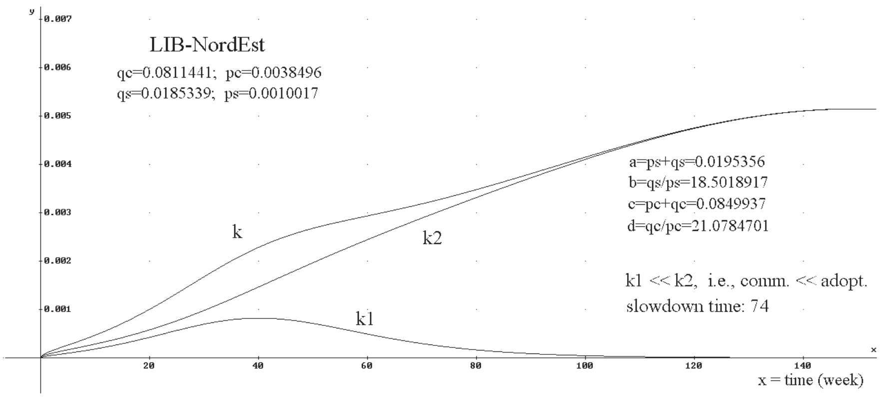

Figure 6: "LIB-NordEst": two synergistic components. Communication (k_1) is a precursor of adoption (k_2) for "LIB" in the "NordEst" area of Italy. Data source: IMS-Health, Italy. Normalised weekly packages sold; period: 8/2005 – 7/2007.

Table 4: Pharmaceutical drugs' diffusion. Parameter estimates of co-evolutionary models for "LIB-NordEst" and "LYR-Italy". t^+_{com}, ${}_Ft_{0.5}$, $\bar{t}_F$ and t^+_{ado}, ${}_Gt_{0.5}$, $\bar{t}_G$ define the modal, median, and mean times of communication and adoption components; the weak order between communication component, $k_1(t)$, and adoption component, $k_2(t)$, is denoted by the symbol "<<".

drug-area	q_c	p_c	q_s	p_s	R^2	t^+_{com}	t^+_{ado}
LIB-NordEst	0.0811441	0.0038496	0.0185339	0.0010017	0.999939	35.9	149.4
LYR-Italy	0.0532225	0.0008988	0.0945056	0.0340769	0.999531	75.4	7.9
drug-area	t^+_{com}	${}_Ft_{0.5}$	$\bar{t}_F$	order	t^+_{ado}	${}_Gt_{0.5}$	$\bar{t}_G$
LIB-NordEst	35.9	36.9	38.1	<<	149.4	154.6	160.3
LYR-Italy	75.4	76.0	77.0	>>	7.9	12.2	14.1

treat. The painful conditions of patients affected by these problems make their continuous search for every possible solution unsurprising. The use of anti-epileptic drugs for neuropathic pain management began with two active compounds, *Carbamazepine* and *Gabapentin*; however, as reported in [14], these drugs did not always have the expected results, so that physicians and patients were waiting for the announced new generation of "neurostabilizer" drugs. Thus, when "LYR" was put into commerce, there probably were patients ready to adopt it; that is, an accumulation of demand prior to product launch. For a similar motivation, see in particular the work by [5], even if the technical approach proposed is based on a dual-market hypothesis. Moreover, "LYR" exhibits a saturating life cycle, which is probably due to

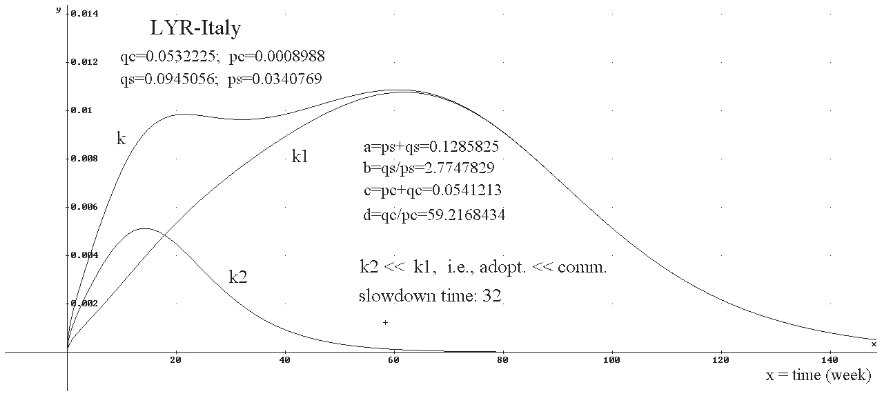

Figure 7: "LYR-Italy": two synergistic components. Adoption (k_2) is a precursor of communication (k_1) for "LYR" in Italy. Data source: IMS-Health, Italy. Normalised weekly packages sold; period: 8/2005 – 7/2007.

its special formulation, based on a cumulative concentration with a delayed response, and to the cost of prolonged therapy. These aspects may explain the reduction in adoptions, with a possible reversion to *Gabapentin*.

On the other hand, "LIB" does not treat specific pathologies, but is assumed as a precautionary measure to avoid more serious consequences of mild to moderate hypertension, within a large class of competing calcium-antagonists. In this case we may conclude that communication, both institutional and informal, is the main driver of diffusion, and has exerted its effect, stimulating adoptions through the generation of market potential.

In Table 5, we report the corresponding reparameterisation and some information regarding the presence of a slowdown. In the case of "LYR-Italy", we observe a deeper effect, a saddle, indicating a stronger separation between communication and adoption.

Table 5: Pharmaceutical drugs' diffusion. Parameter estimates of co-evolutionary models for "LIB-NordEst" and "LYR-Italy". New parameterisation: a and b refer to adoption, c and d to communication.

drug-area	$a = p_s + q_s$	$b = q_s/p_s$	$c = p_c + q_c$	$d = q_c/p_c$	$k_1 \sim k_2$	slowdown	saddle
LIB-NordEst	0.0195356	18.5018917	0.0849937	21.0784701	$<<$	yes	no
LYR-Italy	0.1285825	2.7747829	0.0541213	59.2168434	$>>$	yes	yes

As a general observation for innovation diffusion, we notice that the "dual-effect" approach recognises levels and locations of two fundamental forces in market expansion. Our examples seem to suggest that the difference in behaviour may depend on the degree of innovation contained in the new product. Radical innovations, dominating their own market niche and often patented, may exhibit an interesting inversion -adoption/communication- which would require actions oriented towards specialised agents. Vice versa, incremental innovations, based on an already existing technology and competing in a mature marketplace, may be treated with a more common managerial effort based on continuous and diffuse communication actions.

5. Final Remarks and Discussion

This paper examines theoretical, technical and applied aspects of a well-known diffusion-of-innovations class of effects: slowdown, dip, saddle or chasm. The proposed dual-effect hypothesis emphasises a new interpretation of this systematic depression in the early stages of the diffusion process. This effect may be captured by a binary model for an adoption process nested in an evolving communication network, namely a precursor of the corresponding market potential (see [10]).

However, there is incontrovertible evidence concerning the non-uniqueness of the causal forces generating a slowdown. Dual-market interpretations or correlated mixture modelling representations introduce a two-segment partition of adopters assuming a temporal decomposition of adoptions pertaining to rigid segments. Nevertheless, repeated adoptions due to the same adopter may be realised in different contexts and with different awareness levels.

In the sequel, we summarise some specific properties and limitations of the dual-effect approach:

a) The decomposition of the density related to the co-evolutionary model highlights the presence of a self-reinforcing diffusion governed by two synergistic forces, communication and adoption, that are not ordered in a fixed way during time evolution

b) In particular, an interchangeable allocation of the two driving forces is possible. As we have observed by examining two new pharmaceutical drugs introduced in Italy in 2005, for one of them there is an "inversion" in the role of adoption, which may be interpreted as a consequence of the severity of the pathology and the accumulated demand effect in the initial stages of diffusion

c) The alternative order is simple to detect with a strong *likelihood ratio order* between $k_1(t)$ and $k_2(t)$ or, much more practically, through a simplified *weak order* between $f(t)$ and $g(t)$ based on easy to compute location indexes, i.e., mode, median, and mean values

d) The model only requires time series of adoptions. In particular, the dynamics of market potential, which take into account the evolution of the communication effort and related word-of-mouth, are estimated through the only observable adoption data. Moreover, from a computational and statistical point of view, the proposed framework is easy to implement with common commercial software

e) A slowdown may be originated by an external perturbation, which is a totally different context. If we are able to absorb a local depression with a generalised Bass model (GBM) following, e.g., [15] or [16], then we should adequately motivate or support this modelling choice, which may be correct in some circumstances, but not in others.

Appendix A

The Diffusion of New Drugs in Italy

In this Appendix we report the results of other drugs launched in the Italian market during the period April 2005–August 2007. As will be clear, these results confirm the findings described in the paper.

We examine here the following configurations: "FOL-NordEst", "FOL-Centro", "REX-Italy", "KEP-NordEst", where the first part of the code refers to specific drugs: "FOL", "REX" and "KEP".

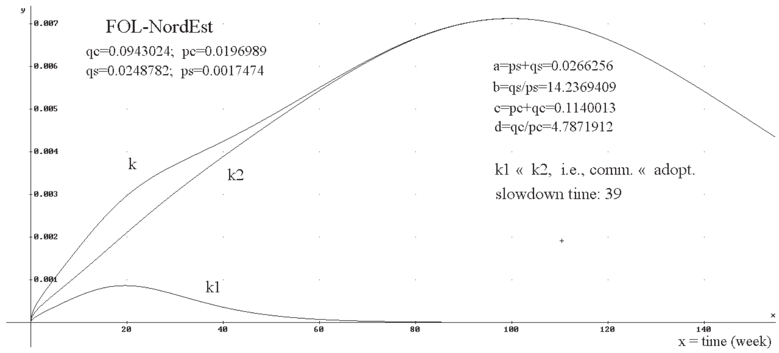

Figure .8: "FOL-NordEst": two synergistic components. Communication (k_1) is a precursor of adoption (k_2) for "FOL" in the "NordEst" area of Italy. Data source: IMS-Health, Italy. Normalised weekly packages sold; period: 8/2005 – 7/2007.

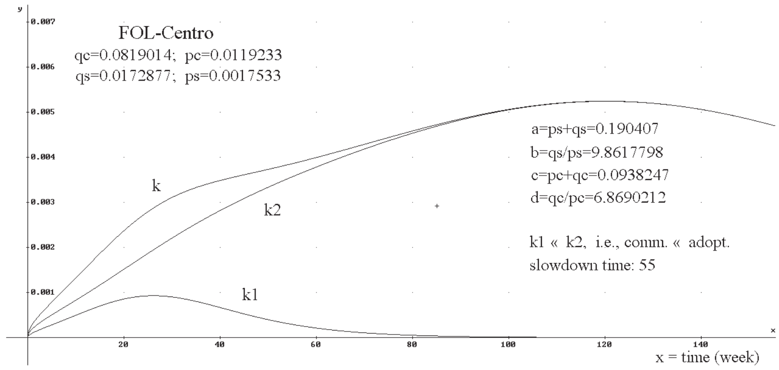

Figure .9: "FOL-Centro": two synergistic components. Communication (k_1) is a precursor of adoption (k_2) for "FOL" in the "Centro" area of Italy. Data source: IMS-Health, Italy. Normalised weekly packages sold; period: 8/2005 – 7/2007.

"FOL" was introduced in Italy in August 2005 to prevent fetus malformations such as exencephaly and neoplasms. Based on *Folic Acid*, this

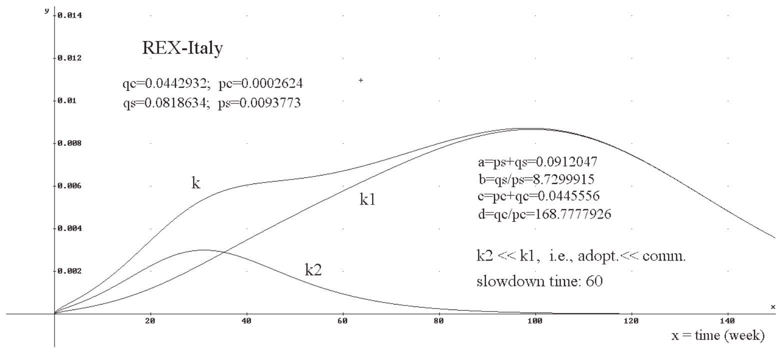

Figure .10: "REX-Italy": two synergistic components. Adoption (k_2) is a precursor of communication (k_1) for "REX" in Italy. Data source: IMS-Health, Italy. Normalised weekly packages sold; period: 8/2005 – 7/2007.

drug is prescribed by physicians to expectant mothers during early gestation. The consumption of Folic Acid by expectant mothers and by women who are planning a pregnancy has been the subject of educational campaigns in many countries.

"REX" was introduced in Italy in August 2005. Its active compound is *Lovastatin*. The most commonly covered diseases are hypercholesterolemia, familial hypercholesterolemia, and hyperlipoproteinemias. Lovastatin is the most recent statin introduced in Italy.

"KEP" was launched in Italy in April 2005. Its new active compound is *Ketoprofen*, commonly employed for treating pain and inflammations.

In Table .6 we summarise the estimation results for the proposed cases under a standard nonlinear least squares approach (Levemberg-Marquardt; see, for instance, [12]) in the co-evolutionary model as expressed in Equation (17).

We may notice from the values of the determination index, R_1^2, that the models present very high levels of global fitting and all the involved parameters are significant.

As already observed in [10], for "FOL" the communication parameters have a higher value in the area of "NordEst", while they are much lower in "Centro", despite the apparently better diffusion process in this area (see asymptotic market potential K).

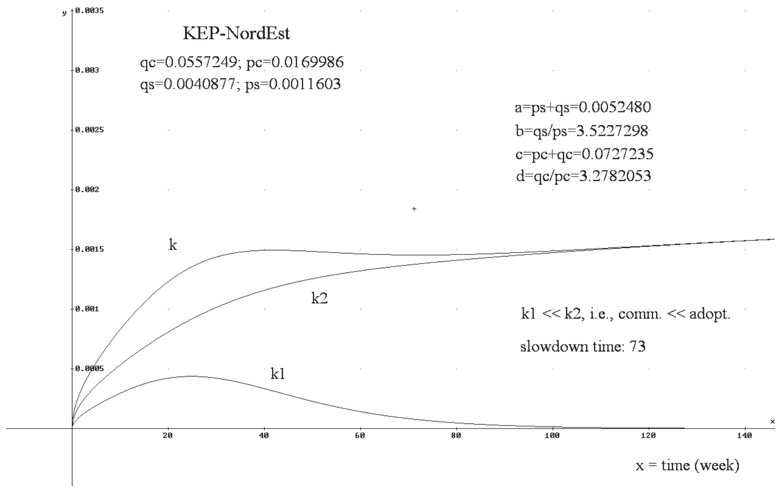

Figure .11: "KEP-NordEst": two synergistic components. Communication (k_1) is a precursor of adoption (k_2) for "KEP" in the "NordEst" area of Italy. Data source: IMS-Health, Italy. Normalised weekly packages sold; period: 8/2005 – 7/2007.

In both cases, the presence of a slowdown in data evolution strongly departs from the classical bell-shaped Bass model (BM). This effect is quite perfectly absorbed by the model.

"REX-Italy" presents good behaviour at the national level with an excellent NLS fitting, $R_1^2 = 0.999957$, and its instantaneous evolution emphasizes a pronounced slowdown.

"KEP" denotes an acceptable behaviour in the "NordEst" area with a good NLS fitting, $R^2 = 0.99964$. We notice a slowdown generating a proper saddle effect.

In Table .7, we summarise the basic estimates, the modal time values, t^+_{com}, t^+_{ado}, the median time values, ${}_Ft_{0.5}$, ${}_Gt_{0.5}$, and the mean values, $\bar{t}_F$, $\bar{t}_G$, referred to communication and adoption components, respectively.

Observing Figures .8, .9, .11, we see that in three cases, namely "FOL-NordEst", "FOL-Centro", "KEP-NordEst", the weak order based on location indexes confirms a predictable behaviour, according to which communication has a driving role, preceding and pulling adoptions. In contrast, in the case of "REX-Italy" depicted in Figure .10, we observe an explicit inversion, so that the adoption component has a driving role in the first part of diffusion.

As we anticipated in Section 5, an early growth of adoptions is to be expected in the case of drugs for severe pathologies.

Table .6: Pharmaceutical drugs' diffusion. Parameter estimates of co-evolutionary models for "FOL-NordEst", "FOL-Centro", "LIB-NordEst", "REX-Italy", "KEP-NordEst", "LYR-Italy" areas of Italy. ( ): marginal linearised asymptotic 95% confidence limits

K	q_c	p_c	q_s	p_s	R_1^2	*indexes*
"FOL-NordEst"						
339352	0.09430	0.01969	0.02487	0.00175	0.999961	$DW = 0.556$
(320070)	(0.07664)	(0.01657)	(0.02385)	(0.00170)	$SSE:$	$P^2 = 0.9352$
(358633)	(0.11196)	(0.02282)	(0.02590)	(0.00180)	[8.39324E6]	$F \simeq 671$
"FOL-Centro"						
763867	0.08190	0.01192	0.01728	0.00175	0.999967	$DW = 0.476$
(638683)	(0.07342)	(0.01099)	(0.01549)	(0.00154)	$SSE:$	$P^2 = 0.9708$
(889051)	(0.09038)	(0.01229)	(0.01908)	(0.00197)	[1.99934E7]	$F \simeq 1546$
"REX-Italy"						
1748620	0.04429	0.00026	0.08186	0.00938	0.999957	$DW = 0.328$
(1664410)	(0.04288)	(0.00025)	(0.07727)	(0.00889)	$SSE:$	$P^2 = 0.9776$
(1832830)	(0.04571)	(0.00027)	(0.08646)	(0.00986)	[4.24433E8]	$F \simeq 2073$
"KEP-NordEst"						
1652380	0.05573	0.01699	0.00409	0.00116	0.99964	$DW = 0.047$
(–)	(0.03446)	(0.01361)	(-0.00608)	(-0.00385)	$SSE:$	$P^2 = 0.7662$
(9000760)	(0.07699)	(0.02038)	(0.01426)	(0.00617)	[2.80659E8]	$F \simeq 187$

"REX" is based on a new statin for the treatment of hypercholesterolemia. As reported by several studies, statins are the most effective form of treatment for high cholesterol when dietary changes prove insufficient. Although hypercholesterolemia is not a disease *per se*, its correlation with cardiovascular diseases has been widely observed. The fact that "REX" has experienced an early dominance of adoptions would confirm that, for those patients affected by severe hypercholesterolemia, pharmaceutical treatment is a reasonable measure to prevent hard cardiovascular outcomes, such as death or myocardial infarction. This prompt response of patients to the new statin (in the Italian market) is arguably due to previous negative interactions of *Cerivastatin*.

On the other hand, "FOL" and "KEP" are drugs that do not treat specific pathologies but are assumed as a precautionary measure to avoid more serious consequences ("FOL") or as a treatment for minor ailments ("KEP"), so that they present a normal behaviour "first communication, then adoption".

In Table .8 we report the corresponding reparameterisation and some information regarding the presence of a slowdown.

Table .7: Pharmaceutical drugs' diffusion. Parameter estimates of co-evolutionary models for "FOL", "REX" and "KEP" in some areas of Italy. t^+_{com}, ${}_Ft_{0.5}$, $\bar{t}_F$ and t^+_{ado}, ${}_Gt_{0.5}$, $\bar{t}_G$ define the modal, median and mean times of communication and adoption components; the weak order between communication component, $k_1(t)$, and adoption component, $k_2(t)$, is denoted by the symbol "<<".

drug-area	q_c	p_c	q_s	p_s	R^2	t^+_{com}	t^+_{ado}
FOL-NordEst	0.0943024	0.0196989	0.0248782	0.0017474	0.999961	13.7	99.7
FOL-Centro	0.0819014	0.0119233	0.0172877	0.0017533	0.999967	20.5	120.2
REX-Italy	0.0442932	0.0002624	0.0818634	0.0093773	0.999957	115.1	23.7
KEP-NordEst	0.0557249	0.0169986	0.0040877	0.0011603	0.999640	16.3	239.9

drug-area	t^+_{com}	${}_Ft_{0.5}$	$\bar{t}_F$	order	t^+_{ado}	${}_Gt_{0.5}$	$\bar{t}_G$
FOL-NordEst	13.7	16.8	18.6	<<	99.7	104.7	109.5
FOL-Centro	20.5	23.3	25.2	<<	120.2	129.9	138.0
REX-Italy	115.1	115.4	115.9	>>	23.7	26.0	27.8
KEP-NordEst	16.3	22.9	26.1	<<	239.9	325.6	369.2

Table .8: Pharmaceutical drugs' diffusion. Parameter estimates of co-evolutionary models for "FOL", "REX", "KEP" in some areas of Italy. New parameterisation: a and b refer to adoption, c and d to communication.

drug-area	$a = p_s + q_s$	$b = q_s/p_s$	$c = p_c + q_c$	$d = q_c/p_c$	$k_1 \sim k_2$	slowdown	saddle
FOL-NordEst	0.0266256	14.2369409	0.1140013	4.7871912	<<	yes	no
FOL-Centro	0.0190407	9.8617798	0.0938247	6.8690212	<<	yes	no
REX-Italy	0.0912407	8.7299915	0.0445556	168.777793	>>	yes	no
KEP-NordEst	0.0052480	3.5227298	0.0727235	3.2782053	<<	yes	yes

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