How much Natural Gas is there? Depletion Risk and Supply Security Modelling

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Abstract

Natural gas supply security and depletion risks highlight a parallel framework similar to crude oil evolution in extraction and consumption. We model this process at a global level. A local comparison is introduced with reference to Former Soviet Union and Italy. Modelling is based on the generalized Bass model (GBM) with special interventions in order to represent and interpret historical large perturbations. The combined use of nonlinear least squares (NLS) and autoregressive moving average with transfer functions of regressive variables (ARMAX) allows a good inference performance starting from British Petroleum (BP) production data. The main results refer to peaks and depletion times estimation under both constant and modified scenario hypotheses.

Key words:
Natural gas depletion, Diffusion process, Generalized Bass model, Nonlinear models, Oil depletion, ARMAX

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1 Introduction

- Nomenclature -

Natural gas is one of the cleanest, efficient and safest energy sources and it has a variety of uses in industry, agriculture, transport and heating. It is normally associated with oil reservoirs underneath the earth or in deeper deposits when it stands alone. In its pure form it is colorless and odorless with a high energy density. Unlike other fossil fuels, natural gas combustion emissions are quite limited with low impact on health and environment. Natural gas is a mixture of hydrocarbon gases: methane (CH_4) $80-95\%$, ethane (C_2H_6) $5-15\%$, propane (C_3H_8) and butane (C_4H_{10}) $< 5\%$. In its natural or *wet* form it contains low fractions or traces of rare gases like Helium, Argon, Xenon, Neon and Carbon dioxide, Nitrogen, Hydrogen sulphide and other residuals (water, sand, etc.). The *dry* version is essentially pure methane and is obtained with special separating processes that remove heavier hydrocarbons, rare gases, water, sand and other residuals. The clean natural gas is transported through a network of pipelines of large diameter and then, after appropriate pressure standardization, it is delivered to the points of use through small diameter local networks. Natural gas can be measured in volumetric or thermal units. Production industry normally measures natural gas in billions of cubic feet (bcf) or trillion of cubic feet (Tcf). In Europe a common measure is in billions of cubic meters (bcm): 1 bcm corresponds to 35.3 bcf. As a source of energy, natural gas is commonly measured in British thermal units (Btu): a cubic foot corresponds to 1027 Btus.

- Origins -

There are different theories on the origin of fossil fuels in general.

The most widely accepted for natural gas origin is the *thermogenic methane theory* which is directly related to crude oil formation. The organic matter, microorganisms, plants, animals, etc. lived millions of years ago was combined with mud, sediments and compressed under crustal movements. Compression and high temperatures underneath the earth broke down the carbon bonds in the organic matter. At low temperatures crude oil were the common outcome of such transformations. With an increasing depth the corresponding higher temperatures originated more natural gas. Mixed natural gas and oil reservoirs are located 1 to 3 kilometers under the earth crust. Deeper underground deposits normally contain natural gas and sometimes pure methane.

A different theory, the *biogenic methane theory*, is based upon the anaerobic transformations of organic matter by microorganisms near the earth's surface. Under rare circumstances this methane, normally dispersed into the atmosphere, could had been trapped underground by crustal movements. A well–known current biogenic methane production is landfill gas.

A third theory regarding methane formation is *abiogenic* in nature and is not widely accepted. It is based on the combination of hydrogen–rich gases with carbon molecules in the absence of oxygen at a high depth under the earth's crust. These combined gases move towards the surface of the earth forming methane deposits in reservoir sites.

The most relevant differences among the previous theories refer to time origin of the natural gas formation, to the localization and to the physical extension. The *thermogenic methane theory* does not exclude present new formation of methane but in mankind time interval these contributions are irrelevant as compared with the originating bio–physical processes dated millions of years ago. This is the reason why natural gas, like other fossil fuels, is considered as a non–renewable resource.

- *Trapped natural gas* -

Most of the natural gas rose (and rises today) to the earth surface and dissipated into the atmosphere. A limited deal rose up into special geological formations characterized by layers of porous sedimentary rocks covered by an impermeable layer of denser rocks. These special geological formations, *anticlinal formations*, trapped natural gas and oil under the ground as their extent is sufficiently large. They constitute reservoirs that may be technically and economically exploited. Extraction of natural gas is based on drilling rigs plants that allow the release of natural gas and, possibly, of oil under natural or artificial pressure. We do not describe here "unconventional" formation of natural gas deposits.

- *A brief history* -

Natural gas first historical track is recognized in the ancient Greece on Mount Parnassus in about 1000 B.C., where an astonishing *burning spring* rising from a fissure in the rock was interpreted as a sign of the divinity. A temple in this site housed a priestess known as the Delphi Oracle. Claimed prophecies were inspired by the flame. Five centuries later, in about 500 B.C., the Chinese discovered a way to transport natural gas with bamboo canes from fissures in the earth. It was used to obtain drinkable water and salt from sea water by a boiling process. Britain commercialized "natural gas" (coal gas) around 1785 to light houses. In 1821 William Hart dug the first gas well in Fredonia, New York (27 feet under the surface of the earth), and successively formed the Fredonia Gas Light Company which is the first American gas company. In 1859 "Colonel" Edwin Drake dug the first oil and natural gas well at 69 feet under the surface level. During the 19th century, natural gas prominent use was lighting and, at the end of the century, with the diffusion of electricity, natural gas lights were substituted by electric bulbs.

The natural bottle–neck that limited a rapid diffusion of natural gas for heating, cooking and industrial transformations was the absence of an efficient pipeline infrastructure.

In 1885, Robert Bunsen invented a special burner (Bunsen burner) that allowed a controllable mixed proportion between natural gas and air for a safety flame use. This invention opened a new way to the use of natural gas in America and anywhere in the world.

The lack of an efficient pipeline network before World War II negatively conditioned the extraction of natural gas when formed alone and, when discovered in mixed form with oil (or coal), was vented into the atmosphere or directly burned in place.

In this period the use of natural gas for transport was limited in general with some exceptions, for instance, in Italy.

Pipeline infrastructure started in 1920 but the effective *take off* is situated after the World War II when new metallurgical advances and new welding techniques allowed the construction of reliable pipelines. The pipelines boom lasted into the 60's in America and some decade later in Europe. The new situation allowed the diffusion, with large scale efforts, of new heating appliances, water heaters, boilers, oven ranges. Industry introduced natural gas in manufacturing and processing plants. In particular, a large diffusion of gas turbine emerged in electricity generation. More recently the diffusion of methane in transport is increasing even if petrochemical industry and governments gave incentives for the commercialization of an heavier fuel, the liquified propane gas, LPG, by constructing, for instance, a good network in Europe.

- *Resources and Reserves* -

The natural gas abundance assessment at global and regional level is relevant because this resource is non–renewable: its formation takes possibly millions of years. Measuring natural gas in the ground is a very difficult purpose because technical, economic, strategic and social viewpoints are partially conflicting or at least not independent from each other.

A great deal of inference and estimation is involved and new technologies are becoming increasingly efficient and reliable. Nevertheless, it is a common opinion that no one really knows exactly "how much natural gas is there" until it is extracted. A confirmation of the conflicting interests and corresponding "technical definitions" regarding different typologies in natural gas resources is recognizable by observing the different classifications systems adopted by Energy Information Administration (EIA), International Energy Agency (IEA), Oil and Gas Journal (OGJ), World Oil magazine (WOm), BP Statistical Review of World Energy (BPSR), World Oil and Gas Review (WOeni).

Let us examine, for instance, the EIA's classification system for natural gas resources. The "Natural Gas Resource Base" (NGRB) is the broadest classification and includes the entire volume of natural gas *contained* and *trapped* in the earth before any extraction. A large part of NGRB is technically non–recoverable with present or near future technologies. In this sense *Conventional natural gas* which may exist in the earth and is trapped in reservoirs is contrasted with *Unconventional natural gas* that takes different forms and is more difficult to extract for a lack of adequate technologies or for too high costs. There are six main categories of unconventional gas: deep natural gas, tight natural gas, Devonian shale gas, coalbed methane, geopressurized zones gas, methane hydrates. The challenge or hope is that as technology advances, the resource potential of unconventional natural gas may be less virtual. The NGRB *Recoverable* resources for which a production technology exists are subdivided into *Discovered* and *Undiscovered Technically Recoverable Resources.* Discovered recoverable resources include up to date production and the gas that remain to be produced, i.e., the "reserves". *Economically Recoverable Resources* are a subset of previous "reserves" for which there are positive economic return on the basis of current market conditions. This threshold may vary along time. The "reserves" which are jointly economically recoverable are broken down in two main classes: *proved reserves* and *other reserves.* Proved reserves are estimated with a 90% probability (P90) and their extraction at present conditions is "reasonably certain". Other reserves are denoted with a variety of terms: probable reserves, possible reserves, indicated reserves, inferred reserved, etc. and are characterized by different levels in extraction probability with a lack of agreement in numeric assessment (P50, P10, P05, etc.).

Table 1
EIA and NPC compiled estimates

US Natural Gas Technically Recoverable Resources (EIA, 1 Jan 2000, Trillion cubic feet)		US Natural Gas Resources (NPC, 1 Jan 1999, Trillion cubic feet)	
Non associated undiscovered gas	247.71	Old fields	305
Inferred reserves	232.70	New fields	847
Unconventional gas recovery	369.59	Unconventional	428
Associated-dissolved gas	140.89		
Alaskan gas	32.32	Alaskan gas (old fields)	32
Proved reserves	167.41	Proved reserves	167
Total Natural Gas	1190.62	Total Natural Gas	1779

Comparing EIA 1 Jan 2000 compiled estimates and the corresponding ones by National Petroleum Council (NPC) in 1999 for the USA, Table 1 denotes the variability of definitions and estimates regarding natural gas supplies and this is an obvious source of uncertainty so that a "definitive" assessment of the residual economically and technically recoverable natural gas is not clearly recognized.

New technologies such as seismic exploration (onshore and offshore), magnetometric and gravitometric satellite tools, 2-D, 3-D and 4-D seismic imaging allow a better understanding and positioning of possible reserves. Revisions of previous estimates are a common consequence. Historical Proved natural gas reserves are generally not changed significantly over past 10–20 years and this may be surprising.

- *Ultimate Recoverable Resource Estimation* -

We consider more convenient an *indirect answer* to the main question involved in present paper's title. Our main interest is in the *Ultimate Recoverable Resource* (URR) determination. Following [1] we may consider NGRB at global or local level as the existing Physical Resource of interest. The corresponding URR for natural gas is only a subset of it. "Direct" determination of a realistic notion of "reserve" must be algebraically discounted with obvious technological, economic and strategic conditions and opportunities so that the URR for natural gas is the most relevant quantity of interest, i.e., the "total amount of a finite resource which may be obtained at the end of extraction or production process" as a result of all concurring forces. This avoids unrealistic efforts based on purely virtual notions. Historical production data summarize the variable joint contributions of technological, economic and social effects, including dynamic learning, on a production of a finite resource interpreted within a *diffusion of innovation* framework under possible exogenous interventions.

A quite similar point of view was expressed in [2] and in [1] with reference to crude oil. The generalized Bass model (GBM) (see Ref. [3]) is the main tool and is combined with an ARMAX framework for the implementation of residual deviation not recoverable with the direct control of a special intervention function that incorporates historical shocks of different origins.

The well-known pioneer of historical forecasting on crude oil depletion is Hubbert [4]. He correctly estimated in 1956 the USA oil peak around 1970 and successively in [5] recognized the equivalence of his model with logistic Verhulst equation (see Ref. [6]). More recently Campbell and Laherrère published in (1998) a well-known paper (see Ref. [7]) on this topic that influenced and stimulated a variety of research on this theme and related issues. Many successive modelling approaches may be thought as extensions of Hubbert logistic basic model. See, for instance, Reynolds [8] and [9] for price an cost inclusions, Laherrère for creaming curves, multi–Hubbert modelling [10], [11], [12] and reserves/production analysis. Previous approaches, even if correctly based on production data modelling, are not able to include in the model shocks and strategic, regulatory or technological interventions with flexible and interpretable tools.

GBM modellig (see Ref. [2], [1]), possibly combined with appropriate ARMAX sharpening, is a robust and simple method for innovation diffusion modelling and forecasting under selected scenario hypotheses.

The paper is organized as follows. In Section 2 we summarize briefly the technical bases of GBM with the motivations that justify our preference. In Section 3 we study the evolution on world natural gas production. In Sections 4 and 5 we examine two local aspects: Former Soviet Union (FSU) and Italian natural gas productions. The aim is to evaluate, with comparative scenarios, the relationships among peak times and duration estimates. Section 5 is devoted to final remarks and discussion.

2 GBM diffusion model and statistical aspects

A basic differential approach on innovation diffusion modelling is developed by Bass in [13] and his model includes the logistic equation by Verhulst [6] as a special case. An extension, the generalized Bass model, GBM, introduced by Bass, Krishnan and Jain (see Ref. [3]), incorporates exogenous intervention factors that control evolutive dynamics.

There are different ways in diffusion of innovation modelling. Complex System Analysis, CSA, (see, for instance [14], [15], [16] and [17], among others) allows a micro foundation basis of aggregate behaviour. Stochastic differential approach or a semi–deterministic differential approach are intimately connected with CSA (see [18] and [19]) and are more suitable for statistical aggregate data handling when we have to learn from an existing life cycle process during its evolution.

The diffusion of innovations in a social system is represented over time by a status change of an adoption unit from the *neutral* state to the *active* state. Within the standard Bass model [13] the system of potential adopters, or reference population, is decomposed into three sub-populations: *innovators*, *imitators* and *neutrals*. For simplicity reasons we may consider a one-to-one correspondence between single adoption and corresponding adopter. Each subpopulation is characterized by a separate share: p, q and respectively, $1-p-q$.

The conditional adoption probability is 1 for innovators, z/m for imitators (word–of–mouth) and 0 for neutrals. The typology of a potential adopter is unknown so that, by a simple marginalization, we obtain the corresponding *hazard ratio*, i.e.,

$$\frac{z'}{m-z} = p \cdot 1 + q \cdot \frac{z}{m} + (1-p-q) \cdot 0, \tag{1}$$

that describes the adoption probability of a generic unit belonging to the non adopters group, i.e., the residual market $(m-z)$, where m depicts the potential market (or carrying capacity) and z defines the cumulative adoptions (or adopters) at time t. The instantaneous adoptions in the standard Bass model, BM, are therefore,

$$z' = \left(p + q\frac{z}{m}\right)(m-z) = m\left(p + q\frac{z}{m}\right)\left(1 - \frac{z}{m}\right). \tag{2}$$

The carrying capacity, m, is not exogenously defined. It is simply a special unknown parameter of the diffusion process. For example, the potential market is only a subset of a susceptible population of consumers and, as we will see in Section 3, natural gas URR is only a subset of the corresponding Physical Resource.

The standard Bass model is extended in [3] with the introduction of a general multiplicative perturbation described by an integrable function $x(t)$ representing political, economic and strategic interventions. Its neutral level, $x(t) = 1$, depicts the standard Bass model, BM.

The GBM is then

$$z' = m\left(p + q\frac{z}{m}\right)\left(1 - \frac{z}{m}\right)x(t) = \left(p + q\frac{z}{m}\right)(m-z)x(t), \tag{3}$$

and the general closed form solution is

$$z(t) = m\frac{1 - e^{-(p+q)\int_0^t x(\tau)d\tau}}{1 + \frac{q}{p}e^{-(p+q)\int_0^t x(\tau)d\tau}} = mF(t), \qquad 0 \le t < +\infty. \tag{4}$$

Function $F(t)$ is a modified Riccati distribution function (see Ref. [20]). The function $x(t)$ modifies the geometry of time, and not the carrying capacity, m, or the intrinsic diffusion parameters p and q that are dimensionally independent. It can be easily proven (see Guseo [21]) that asymptotic quotas of innovators and imitators are not affected by $x(t)$.

Under the assumption that memory effects have a non–uniform distribution over time, we can model the intervention function $x(t)$ in Equation (3) through some different shocks, for example, exponential and rectangular, i.e.,

$$x(t) = 1 + c_1 e^{b_1(t-a_1)} I_{t \geq a_1} + c_2 I_{t \geq a_2} I_{t \leq d_2}, \qquad a_2 < d_2. \tag{5}$$

In the exponential case c_1 controls depth and sign of perturbations, b_1 describes effects persistency over time and a_1 denotes the starting time. Note that usually parameter b_1, is negative if memory is decaying to the stationary position (mean reverting), i.e., $x(t) = 1$. Sometimes it may be positive and this aspect introduces a strong acceleration in the saturation of a life–cycle. In the rectangular case, c_2 controls depth and sign of perturbations, a_2 and d_2 define origin and ending points of a locally stationary intervention.

Function $x(t)$ may be defined with the implementation of exogenous variables, i.e., price variations, marketing mix variables, regulations, policies, etc., in order to test their effects.

Adoptions may also be anticipated or delayed randomly so that it is natural the inclusion in model building of stochastic residual components.

A nonlinear regressive specification of a statistical version of a GBM is,

$$y(t) = f(\beta, t) + \varepsilon(t) = z(t) + \varepsilon(t), \tag{6}$$

where the deterministic part $f(\beta, t)$, equal to $z(t)$, is a nonlinear function of the unknown vector of parameters $\beta \in R^k$ partially included in $x(t)$. The component $\varepsilon(t)$ is a stochastic process representing the i.i.d. residual error. The usual regressive assumptions consider $\varepsilon(t)$ as a white noise process, possibly based on a normality assumption.

Therefore, the estimate $\hat{\beta}$ of β parameters in Equation (6) are determined by a non linear least squares procedure (e.g. Marquardt, Gauss–Newton or other criteria. See for instance Seber and Wild [22]). In a second phase, we examine the estimated residuals of nonlinear regression, i.e.,

$$\hat{\varepsilon} = y(t) - f(\hat{\beta}, t).$$

If residuals do not support the hypothesis of a white noise process it is possible to consider a well-known autoregressive moving average transfer function model (ARMAX), proposed by Box and Jenkins [23] in order to approximate dynamic relationships between an input x_t and an output z_t, see for instance [21].

In ARMAX, or, more generally, in ARIMAX models we can explain variable $z_t = y(t)$ not only with delayed autoregressive (AR) or moving average (MA) components but with the help of a function $g(\cdot)$ of the input controlling variable, possibly with delayed terms, which in our case is directly described by the average behaviour $f(\beta, t)$. More formally we have

$$\Phi(B)\nabla^d\{y(t) - g[f(\beta, t), \ldots, f(\beta, t-k)]\} = \Theta(B)a_t, \tag{7}$$

where $\Phi(B)$ and $\Theta(B)$ are backward polynomial operators of order p and, respectively, q, ∇^d is a difference operator of order d, a_t is a white noise, $a_t \sim WN(0, \sigma^2)$, and $g(\cdot)$ is the transformed regressive component.

Inference in Equation (7) may be performed with a sub–optimal two–steps (or more steps) procedure with some simplifying assumptions. Within the applications described in Sections 3–5 we consider, for instance,

$$\Phi(B)\{y(t) - \alpha f(\beta, t)\} = \Theta(B)a_t, \tag{8}$$

or

$$\Phi(B)\{y(t) - \alpha_1 f(\beta, t) - \alpha_2 f(\beta, t-2)\} = \Theta(B)a_t. \tag{9}$$

At a first step we estimate the transfer function $f(\hat{\beta}, t)$ within a regressive framework and then, at a second step, we optimize ARMAX models in Equations (8) and (9) conditionally on $\hat{\beta}$. The lack of fit of this step–wise procedure is however quite limited.

In general, it is not a good practice to build a model in which a global optimizer determines simultaneously the most important components if we perceive a hierarchy among model components. Actually, this fact may generate a local good fitting with confounding effects between evolutive deterministic interventions and stochastic components.

In particular, in GBM modelling there is a prominent role of *natural diffusion* which provides a start–up of the process (innovative component) and iteratively initializes the imitative or logistic component which assures convenient curvature before saturation. Usually interventions may have a more limited space.

In fact, a second level in the model construction is necessarily devoted to the deterministic identification of systematic perturbation $x(t)$. Such perturbations may sometimes strongly interact with the *natural diffusion* by shifting peaks, delaying or anticipating saturations. This is driven by the real process.

A third level refers to the stochastic components. In some situations, as we can see later on in Sections 3–5, such components may have a relevant role.

Finally, some attention must be paid to the nonlinear inference with a large set of parameters. Initializing choices, interruption rules and convergency tests are not simple matters and require some joint statistical evaluation of specific inference criteria especially with reference to the marginal linearized asymptotic confidence intervals which are not simultaneous in nature.

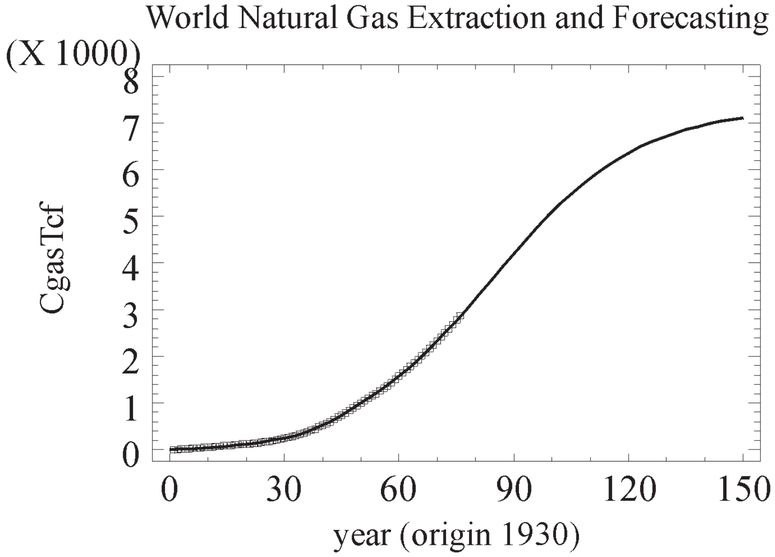

Fig. 1. World natural gas cumulative production: One shock GBM model.

3 World natural gas production evolution

The World natural gas production is examined in detail in order to estimate its evolution. We consider a time series related to a recent Campbell's study in (2002) (see Ref. [24]) which covers the period 1930– 2000. The gas data are expressed in Tcf and depict the annual production. We compare these data with the corresponding ones from the BP Statistical Review (2006) (see Ref. [25]) in order to assess possible deviations within the common 1970–2000 time period. Both series are essentially equivalent with minor discrepancies so that we extend Campbell's series by adding BP's data concerning 2001–2005 period after a natural transformation from daily BP production data to annual corresponding values.

A parallel evolution between crude oil and natural gas consumptions may be considered a reasonable hypothesis with some differences related to network

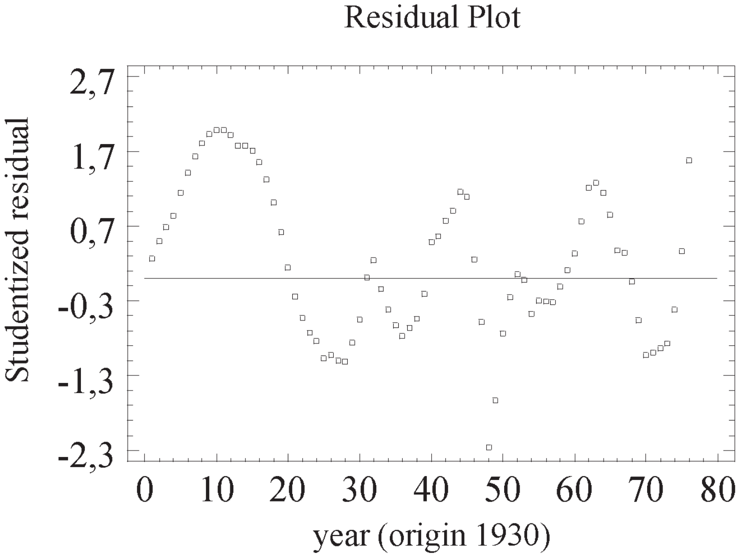

Fig. 2. World Gas Cumulative Production: Studentized residuals in One shock GBM model.

externalities. The inspection of instantaneous data production, see Figure 3, confirms a positive systematic increment in natural gas production during the post World War II period, in particular, 1960–75. This is quite natural if we compare this hypothesis with the significant positive and permanent exponential shock which originated in 1951 in corresponding world oil data (see Ref. [1]). Nevertheless, we have mentioned in Section 1 that the lack of an efficient pipeline network negatively conditioned the extraction of natural gas from deep reservoirs and, when discovered in mixed form, was vented into the atmosphere or burned in place. Current data do not register correctly this extraction especially in the first post World War II period.

Therefore, we consider a GBM with a rectangular shock properly designed for the absorption of this hypothetical systematic deviation which is strongly correlated with the international changes in life styles after World War II with previously mentioned omission in data registration.

The obtained results are quite satisfactory. See, in particular Table 2.

Determination index has a particularly high level, $R^2 = 0.999965$. Figure 1 confirms this situation for the estimated cumulative framework. The marginal linearized asymptotic 95% confidence intervals are very small. The ratio $q/p = 108.4$ corresponds (see Ref. [21]) to an asymptotic fraction of innovators very limited, $F_p(\infty) = 4.5\%$. The world diffusion (consumption) of natural gas is essentially, even if not exclusive, an imitative process, but less imitative than oil (see Ref. [1]). We note that the supposed high consumption regime is

Table 2
Parameters estimates of World natural gas cumulative production with a GBM under a rectangular shock. ( ) marginal linearized asymptotic 95% confidence limits

m	p	q	c_1	a_1	d_1	R^2	$D-W$
7332	0.00048	0.05198	0.3197	31.7	48.3	0.999965	0.1396
(6942)	(0.00046)	(0.05091)	(0.2951)	(30.6)	(47.8)	SSE:	
(7723)	(0.00050)	(0.05306)	(0.3443)	(32.7)	(48.8)	[1882.57]	

significant and located between year 1962 and year 1978. The estimated URR is about 7332 Tcf and 90% depletion time is year 2056.

This is perfectly coherent with the post World War II analysis considered in Guseo et al. (2006) (see Ref. [1]) where we observe for oil a permanent structural modification in consumption evolution. The 1978 date acts as change–point and is reasonably correlated with 1979 second negative shock on oil production that was introduced by OPEC in order to optimize returns of supply countries. The limited residuals denote an autocorrelated structure, Durbin-Watson test is very small, $D - W = 0.1396$. This situation is confirmed by Figure 2 where the studentized residuals highlight an evident cyclical pattern.

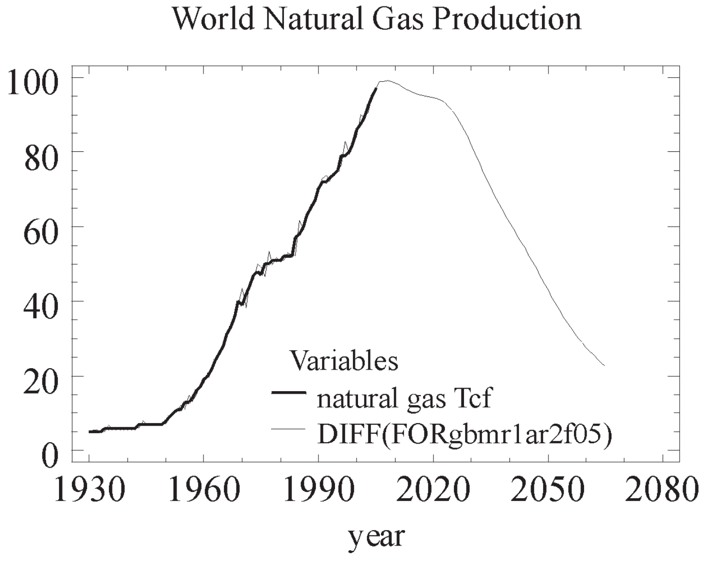

Fig. 3. World natural gas production: One shock GBM model and ARMAX(2,0,0) sharpening.

We apply, as a second step, an ARMAX(2,0,0) representation conditionally on estimated average behaviour, $PREgbmr1f05$, at the first step and under an equilibrium hypothesis regarding the intervention function after 2005, i.e., $x(t) = 1,\ t > 2005$. The main results are outlined in Table 3.

Table 3
World natural gas cumulative production. One shock GBM and ARMAX(2,0,0) sharpening. ( ) t–statistic; [ ] p–values

$AR(1)$	$AR(2)$	$PREgbmr1f05$	$mean$	SSE
1.83265	-0.924146	0.0914463	8.02006	85.05
(32.982)	(-27.1597)	(3.75434)	(2.6492)	$\{d.f.73\}$
[0.000000]	[0.000000]	[0.000347]	[0.009884]	

The proposed ARMAX sharpening is effective. The squared partial correlation coefficient is $\tilde{R}^2 = (1882.57 - 85.05)/1882.57 = 0.9548$ and, in correspondence, the F–ratio, $F = [\tilde{R}^2(N-k)]/[(1-\tilde{R}^2)s] = [0.9548(76-10)]/[(1-0.9548)4] = 348.5$, denote a quite significant improvement. Figure 3 represents observed instantaneous annual natural gas data and corresponding ARMAX(2,0,0) modelling: they are essentially equivalent.

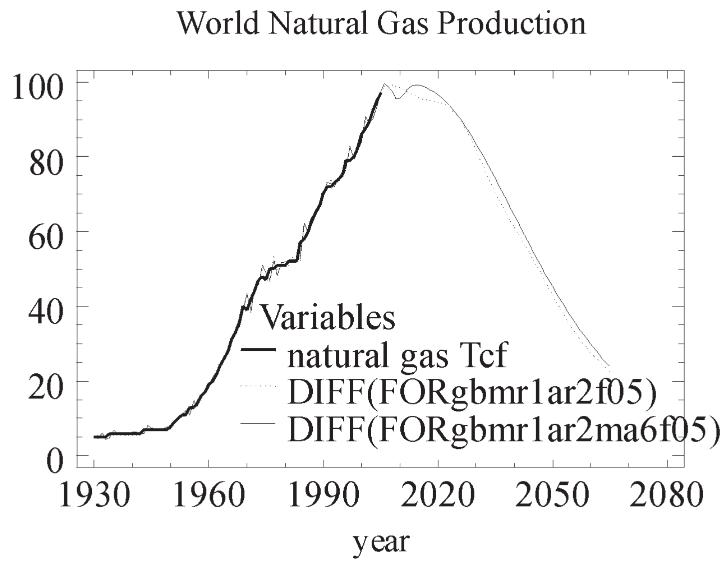

Fig. 4. World natural gas production: One shock GBM model and ARMAX(2,0,6) sharpening.

If we expand ARMAX representation with a more complex model, i.e., ARMAX(2,0,6) we obtain a similar goodness–of–fit. We omit here for brevity reasons the analysis of variance table. Figure 4 represents both ARMAX models. We note that a more complex model originates an oscillatory behaviour on the top with a forecasting tail that equates the simpler model. Natural gas peak is then essentially bimodal and positioned (see Figure 5) between 2008 and 2014. The estimated 90% depletion time, under ARMAX(2,0,6) is year 2052. At that date annual production, under ARMAX(2,0,6) sharpening, is estimated about 41.80 Tcf which may be compared with 2005 observed production, 97.20 Tcf.

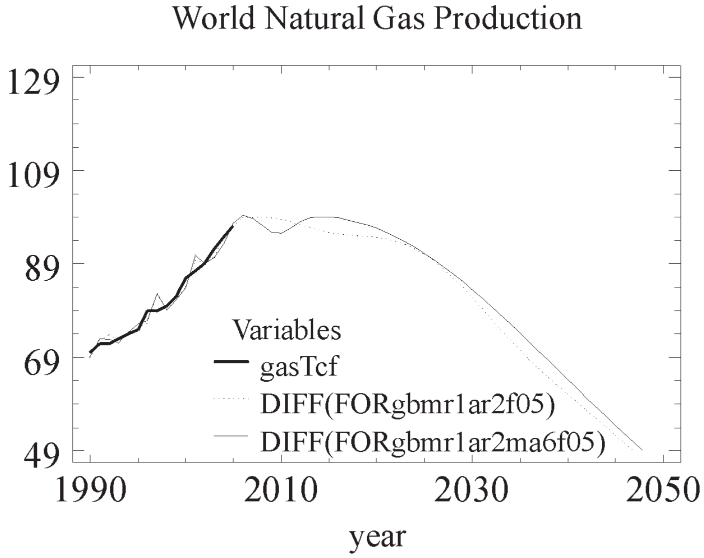

Fig. 5. World Gas Production: One shock GBM model and ARMAX(2,0,6) sharpening (zoom).

The supposed substituting effect of new technologies based on natural gas lasts, if compared with Guseo et al. (2006) results concerning crude oil (see Ref. [1]), 35 years longer with respect to the 90% time depletion level. The estimated peak date is essentially equivalent, 2008, or few years delayed, 2014.

This structural difference in oil and natural gas consumption evolution is not surprising and may be explained with reference to technical, economic, strategic and social forces that acts on these extraction processes. Based on BP 2006 data, we note that international trade movements are about 40% of extracted oil and only 26%, in the natural gas case. The latter is decomposed into two fractions: 74% is transported by pipelines and the remaining part, 26%, is transformed in Liquified Natural Gas (LNG) and successively shipped by dedicated LNG ships. These particular constraints have produced and produce today evident network externalities which are well–known in economics, physics and marketing literature (see, for instance, Ref. [26], [27], [28], [29], [17]). In this sense a possible further examination of this effect could be performed, by applying special modelling as described, for instance, in [19].

4 Former Soviet Union natural gas production

It is well–known that the FSU has a world prominent role in present natural gas production and a better positioning with reference to the “proved reserves”

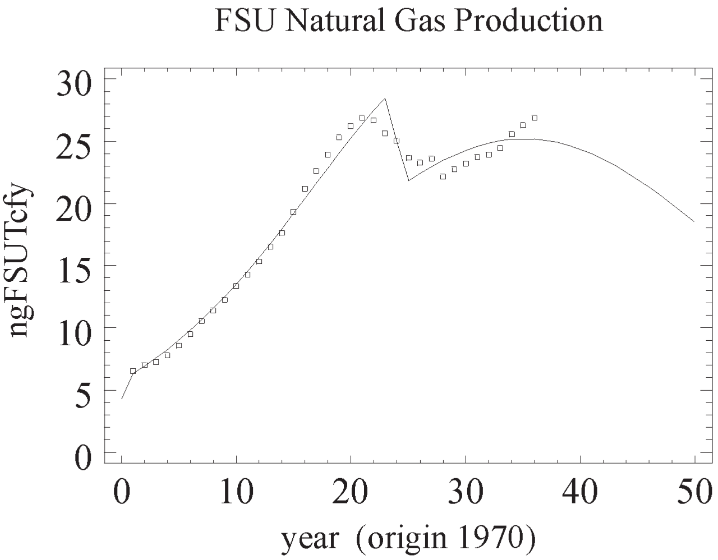

Fig. 6. FSU natural gas production: two exponential shocks GBM model with unknown origin.

in the next decades. If we examine BP Statistical Review 2006 data (see Table 4) we note that 27.5% and 32.4% are the corresponding quantitative values. The natural question is not based on quota level but on an assessment of the quality of "proved reserves" estimates. We discuss this point in an appropriate Section. To this extent let us consider now a specific model for FSU natural gas production.

Table 4
Natural gas production and reserves (2005): Large Areas. Source: BP Statistical Review of World Energy (2006)

	Production (2005)		Proved Reserves (end 2005)	
	Tcf	%	Tcf	%
Of which European Union (25)	19.3	7.2	90.8	1.4
OECD	103.5	39.1	527.7	8.3
FSU	73.6	27.5	2058.8	32.4
Other EMEs	89.3	33.4	3761.6	59.3
	266.4	100.0	6348.1	100.0

In this case we have a shorter time series directly based on BP Statistical Review (2006) data that cover only an advanced portion of the extraction process, i.e., 1970–2005. If we examine Figure 6 we note that the origin of the extraction process must be estimated with a proper parameter c. At time $t = 1$, corresponding to the year 1970, we know only the instantaneous production, i.e., 6.35 Tcf and do not have any direct knowledge of cumulative production till $t = 0$. We suppose that two exponential shocks could be present in the series. A first one with positive effect, c_1, may explain the strong increasing production during the two decades 1970–1990 and a second one, with negative coefficient, c_2, that depicts the significant contraction lasted 7–8 years after 1990. A way to solve the problem at a first step is based on a "density estimation". A very good approximation of Riccati density $f(t) = F'(t)$ is simply

$$f(t) \simeq F(t+0.5) - F(t-0.5). \tag{10}$$

In Table 5 we report the results of this first step nonlinear density estimation.

Table 5
Parameters estimates of FSU natural gas non cumulative production with a GBM under two exponential shocks and a parametric origin. ( ) marginal linearized asymptotic 95% confidence limits

m	p	q	c	c_1	b_1	a_1	c_2	b_2	a_2
1358.9	0.00036	0.0773	28.97	0.3543	-0.014	0.496	-0.348	-0.025	23.97
(373.9)	(-0.081)	(-0.069)	(-2863)	(-1.85)	(-0.259)	(0.4959)	(-0.646)	(-13.0)	(8.81)
(2344)	(0.082)	(0.224)	(2921)	(2.56)	(0.230)	(0.4961)	(-0.049)	(12.9)	(39.1)

Determination index is good, $R_1^2 = 0.982219$ with $SSE = 29.946$ but a large part of marginal linearized asymptotic 95% confidence intervals are too wide with few exclusions: m, a_1, c_2 and a_2. Departures between model and data are rather strong (see Figure 6) and Durbin–Watson statistic, 0.84, denotes prominent residual autocorrelations.

We implement an ARMAX(2,0,0) modelling with a regression component based on previous tentative description, $PREDbasse2t$. The corresponding results are summarized in Table 6

Table 6
FSU natural gas. Two exponential shocks with unknown origin GBM and ARMAX(2,0,0) sharpening. ( ) t–statistic; [ ] p–values

$AR(1)$	$AR(2)$	$PREgbmr1f05$	$mean$	SSE
1.49507	-0.596087	0.077714	6.4137	9.48
(10.03)	(-4.81)	(1.92)	(2.65)	$\{d.f.33\}$
[0.000000]	[0.000032]	[0.064]	[0.0125]	$R_2^2 = 0.9944$

We highlight the partially good performance of the combined extended model. The determination index $R_2^2 = 0.9944$ denotes a noticeable corresponding squared partial correlation index is $\tilde{R}^2 = (R_2^2 - R_1^2)/(1 - R_1^2) = 0.685$ so that the F–ratio, $F = [\tilde{R}^2(N-k)]/[(1-\tilde{R}^2)s] = 11.96$ is fully significant denoting the effectiveness of ARMAX extension. Figure 7 give a good representation of the proposed model. We note the bimodal nature of instantaneous extraction process in 1990 and 2008. After that a smooth declining shape is obtained under an equilibrium hypothesis on intervention function, $x(t) = 1$. URR determination, 1562 Tcf, is based on forecasts summation from 1970 until year 2056, 1470 Tcf, added to an estimated extraction before year 1970, about 92 Tcf $= (6.35 \cdot 29)/2$.

The 90% depletion time, under ARMAX(2,0,0) sharpening, is year 2035. At that date annual production is estimated about 13.30 Tcf which may be compared with 2005 observed production, 26.864 Tcf.

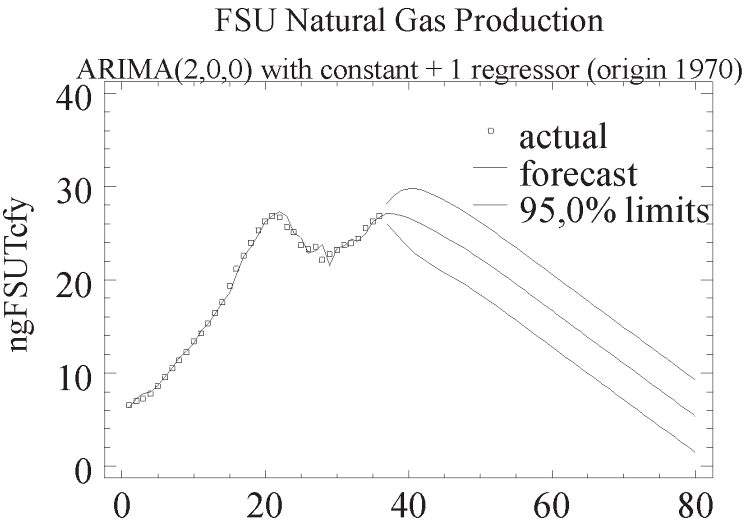

Fig. 7. FSU natural gas production: two exponential shocks GBM model with unknown origin and ARMAX(2,0,0) sharpening.

5 Italian natural gas production

As a third application of our methodology we consider a western state in Europe, Italy, which presents a large dependence in oil and gas supply security with a minor decaying internal natural gas production. Let us consider in Table 7 some information derived from BP Statistical Review 2006.

Table 7
Italy: Natural gas and oil Production, Reserves, Consumption (2005). Source: BP Statistical Review of World Energy (2006)

	Production (2005)		Proved Reserves (end 2005)		Consumption (2005)	
	MToe	%	MToe	%	MToe	%
Oil	6.1	0.2	100	0.1	86.3	2.2
n. gas	10.8	0.4	151	0.1	71.1	2.9
n. gas Tcf/y	0.438		5.9		2.77	

Note, in particular, that Italian oil and natural gas consumption is about 10 times greater than internal production in year 2005. Let us consider now internal evolution of Italian natural gas extraction. In this case we have only a short time series in yearly production, based on BP Statistical Review (2006) data, that cover a recent portion of the extraction process, i.e., 1970-2005. We observe a systematic depression in natural gas production extended from 1973 until 1990. We model such local situation with a parametric origin c and with a rectangular depression delimited by a_1 and d_1 extreme dates within a GBM framework. The obtained results under a nonlinear regressive approach are quite satisfactory (see Table 8). We have added Tcf 4.95 to the cumulative BP series, starting from 1970, as an estimate of previous production.

Table 8
Parameters estimates of Italian natural gas cumulative production. GBM with unknown origin and under a rectangular shock. ( ) marginal linearized asymptotic 95% confidence limits

m	p	q	c	c_1	a_1	d_1	R^2	$D-W$
31.13	0.00379	0.0862	18.75	-0.275	24.6	39.3	0.999919	0.608
(30.26)	(0.00265)	(0.0796)	(16.16)	(-0.302)	(21.6)	(36.6)	$SSE:$	
(31.99)	(0.00494)	(0.0928)	(21.35)	(-0.248)	(27.6)	(42.0)	[0.102645]	

In Figure 8 we can appreciate Italian natural gas cumulative production and its modelling under a shifted origin and one shock included in a GBM model.

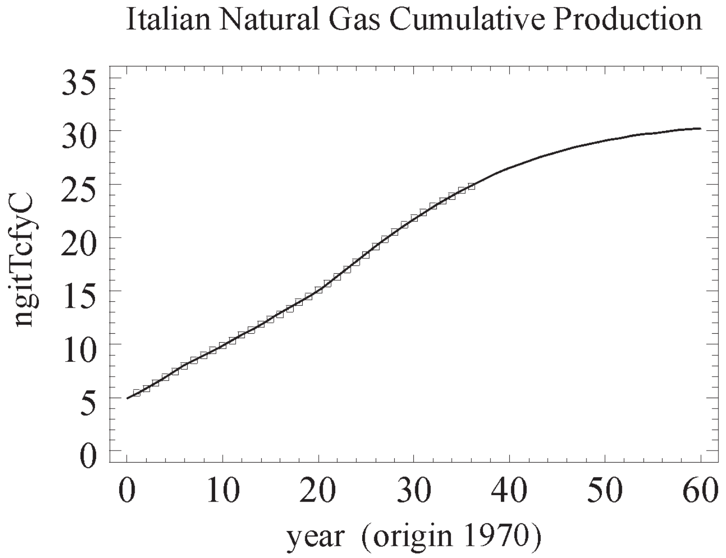

Fig. 8. Italian natural gas cumulative production: One shock GBM model.

The Durbin–Watson statistics, $D-W = 0.608$, suggests the application of an ARMAX sharpening. The preferred solution is based on an ARMAX(3,0,4) with two regressors: the prediction at previous step, $PREbassr1t$, and a lag two delayed prediction, $PREbassr1tm2$. Table 9 summarizes the results with a good SSE, $SSE = 0.03349$.

Table 9
Parameters estimates of Italian natural gas cumulative production. GBM with unknown origin and under a rectangular shock. An ARMAX(3,0,4) sharpening with two regressors

Parameter	Estimate	t	P-value
$AR(1)$	0.891313	9.3485	0.000000
$AR(2)$	0.396363	2.86376	0.007846
$AR(3)$	-0.301844	-3.67383	0.001000
$MA(1)$	-1.09112	-9.85707	0.000000
$MA(2)$	-0.565067	-4.62526	0.000077
$MA(3)$	0.0711969	0.648951	0.521658
$MA(4)$	0.327826	10.4166	0,000000
$PREbassr1t$	0.333831	3.70452	0,000922
$PREbassr1tm2$	-0.322729	-3.29584	0,002669
$Mean$	5.4843	33.0176	0,000000

The estimated squared partial correlation coefficient $\tilde{R}^2$ is particularly high, $\tilde{R}^2 = 0.6737$ and, similarly, the corresponding determination index is $R^2 = 0.999973$. The Italian annual natural gas production, its estimation and forecasting, under an equilibrium hypothesis, $x(t) = 1$, are represented in Figure 9 and confirm a decaying internal production after the observed and estimated peak date, namely 1995. The estimated 90% depletion time is year 2015, under simple GBM, and year 2020, under ARMAX(3,0,4) sharpening. Following the latter model the annual Italian natural gas production in 2020 is about 0.24 Tcf which may be compared with 2005 observed production, 0.44 Tcf.

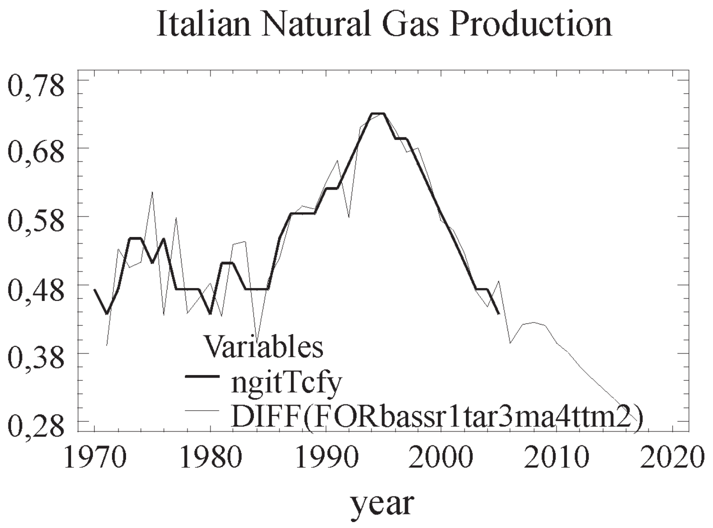

Fig. 9. Italian natural gas production: one shock GBM model with delay and ARMAX(3,0,4) sharpening.

6 Final remarks and discussion

Final remarks and discussion are organized in four thematic groups: 1) methodological aspects; 2) criticism on long–range forecasting; 3) comparison of current results with BP information; 4) FSU reserves divide and correlated interpretations.

As a *first step* we discuss previously some methodological aspects related to Hubbert like approach.

The statistical literature on URR forecasting is quite limited with some important exceptions. Two old reviews in this area are those by Adelman and Jacoby (1979) (see Ref. [30]) and by Kaufmann (1988) (see Ref. [31]). More recent econometric extensions of Hubbert model are provided by Kaufmann (1991) (see Ref. [32]), Cleveland and Kaufmann (1991) (see Ref. [33]) and Pesaran and Samiei (1995) (see Ref. [34]). These extensions are directly based on the logistic Hubbert model for cumulative or rate production (instantaneous) data. There is no attempt to discuss the basic foundations/assumptions of the Hubbert's model. In our terminology this account only for imitative response of the system not for innovative component of the implicit diffusion framework. The proposed extensions for economic, political or technological efforts are not confronted with the definition of differential equation (Verhulst equation modification) wich jointly considers logistic diffusion under an exogenous intervention. This intervention may modify solutions so that much of the oscillatory deviations may be induced by a weakly specified model. In Guseo (see Ref [2] and [1]) we consider, at GBM level, a more elegant solution for intervention inclusion and autocorrelated residuals are examined at a second step by re–estimating the nonlinear mean value transfer function GBM. Recently, a method in comparing logistic (Hubbert model) with gaussian alternative for USA and world oil data are performed in Bartlett (2000) (see Ref. [35]). A general paper on oil depletion by Korpela (2002) (see Ref. [36]) gives a wide insight in the area by considering historical aspects, technological and detailed information regarding the oil actors. Modelling is essentially concentrated in standard Hubbert approach. An interesting section is devoted to various aspects of world oil reserves assessments. Peak dates are determined following translated Hubbert curves proposed by Laherrère. The latter approach is not widespread accepted because it deletes the over production during period 1951–1979 without a compensating equation balancing effect (see, for instance, Guseo [1]). New advances in Hubbert like modelling are provided by Berg and Korte (2006) (see Ref. [37]), where they model simultaneously extended Hubbert models by considering equations systems that jointly account for supply–demand or supply–demand and reserves dynamics. Analysis is performed following qualitative differential equation characterization. There are some difficulties from a statistical point of view related to the direct implementation. In particular, with reference to the definition of proxy variables that represent supply, demand and reserves. An omitted aspect is the description of exogenous effects due to political, economic and social interventions. The inclusion of demand and reserves variables is questionable due to well–known difficulties in availability of reliable information.

A *second focal point* in this discussion is related to expressed criticism in long–range forecasting. Exemplar papers are proposed by Lynch (2002) and (2003) (see Ref. [38], [39]) even if much of his effort is dedicated to naive versions of Hubbert logistic model. More detailed and documented is a paper by Smil (2000) (see Ref. [40]) even if pessimism in scientific work is only a tool for conditional selection of competing theories not a theory in itself. He suggests that "what is immensely more difficult is to anticipate the more likely realities arising from a mix of well–understood and almost inevitable continua on one hand and of astounding discontinuities and surprises on the other". But from this correct evaluation do not follows (*non sequitur*) " that we should abandon detailed quantitative point forecast in favor of the decision analysis or contingency planning under a range of alternative scenarios". In our simple and flexible approach based on a GBM with, possibly, an ARMAX modelling there is a partial and feasible answer to this argument. Intervention function $x(t)$ is historically known and must contribute to the identification and estimation of observed perturbed diffusion of innovation processes. The future is essentially "free" within certain limits and then we can explore, conditionally on some scenario hypotheses, specific forecasts.

A *third aspect* refers to the obtained results and to the corresponding interpretations. In Table 10 we summarize previous assessments for each geographic area in order to allow a comparative comment.

Table 10
Parameters estimates of FSU, Italian and World natural gas production. A comparison with British Petroleum estimated reserves

	Current Model Assessments			BP Information		
	FSU	Italy	World	FSU	Italy	World
URR Tcf	1562	31.1	7332			
Extracted Tcf	782	24.88	2873			
Reserves Tcf	780	6.2	4459	2059	5.9	6348
2005 Production Tcf	26.86	0.44	97.20	26.86	0.44	97.20
Est. ARMAX Prod. in $t_{0.90}$ Tcf	13.30	0.24	41.80			
$t_{0.90}$ GBM		2015	2056			
$t_{0.90}$ ARMAX	2035	2020	2052			

We have motivated in the introductory Section 1 why the currently proposed assessments about “proved reserves” deserve large uncertainties even if measurement technologies are much more precise today. Underground resource measurement is not only affected by natural error measurements. Political, economic and social interactions are combined with technical aspects so that “reserves” must be estimated on the basis of a realistic URR determination. If we compare BP world “proved reserves”, 6348 Tcf, with our estimated URR–dependent reserves, 4459 Tcf, we note a “reasonable reduction” based on the good properties of our data–driven “learning process”. The estimated 90% depletion time at global level under ARMAX representation is year 2052 with a large reduction in annual production, 97.20 Tcf in 2005 vs. 41.80 Tcf in 2052.

With reference to a little producer, Italy, we note that reserves estimates are quite similar, 5.9 Tcf for BP and, respectively, 6.2 Tcf following a GBM modelling. The estimated 90% depletion time, under an ARMAX sharpening is year 2020 with a similar percentage reduction in annual production, 0.44 Tcf in 2005 vs. 0.24 Tcf in 2020.

The most surprising “disagreement” between BP “proved reserves” and our URR–dependent reserves is represented by FSU. BP claimed “proved reserves” are, at the end of 2005, 2059 Tcf, while our model is very “parsimonious”, 780 Tcf. We try to explain such a divide later on. We observe that 90% depletion time under ARMAX representation is year 2035 with a common percentage reduction in annual production, i.e., 26.86 Tcf in 2005 vs. 13.30 Tcf in 2035. In order to evaluate properly previously cited depletion times with corresponding estimated annual natural gas productions, we have to remind that such forecasted values are determined under GBM–ARMAX modelling and, in particular, under an equilibrium hypothesis in intervention function $x(t)$ after year 2005, i.e., $x(t) = 1$. The world and local enormous demand pressure may certainly shrink these conditional, equilibrated scenario, evaluations.

A *fourth remark* is related to a brief discussion of FSU situation in order to interpret previously mentioned divide in “reserves” assessments.

The collapse of the Soviet Union date back to the late 1980s even if formal dissolution refers to December 1991 (Belavezha Accords) with Gorbachev’s resignation (25 Dec 1991).

In 1993 Roseneft was founded and was an insignificant company operating in small oil fields till few years ago. Russia lost much of its global clout with the dissolution of the Soviet Union (after 1990). But after successfully reversing a production slump in the early 1990s, it has re–emerged as an energy superpower. This is an explanation of large depression in natural gas production after 1990 (see, for instance, Figure 6). In 2004 Kremlin broke up his oil company, Yukos. Roseneft nominally acquired Yukos assets from an unknown finance company whose address was a cafè in the city of Tver (see, for instance, Gumbel (2006a) [41]). Russia has large needs of Western capital and there is some hypocrisy in Western fears about Russia's energy sector.

European countries have signed gas contracts with the Russians. For instance, France until 2015 and, more recently, Italy until 2035. "But what happens thereafter that is surely unclear". The natural question is to what extent such contracts (for Italy, France, Germany, etc.) define a realistic basis for gas supply security (in particular for Italy) in the following years. European energy demand is raising while Russia, for its past, needs Western capital if it is to continue rising its oil and gas production (see Ref. [41]).

The second natural question is about the reliability of claimed "proved reserves" which constitute certain guarantees offered as security of capital investments. The pipelines business is an important driver. The Europeans behaviour is contradictory. E.U. governments wants to make energy policy *themselves* and not under the European Commission external energy policy. There are some excellent exclusions, e.g., Poland. Former Chancellor Gerard Schroeder, head of supervisory board between Gazprom (the monopolistic owner of Russian natural gas) and two German firms, is involved in a new underwater Russia–Germany pipeline that avoids to pass through Poland. At the same time, Transneft (the monopolistic Russian pipelines owner) acquired a Russian stake in Caspian Pipeline Consortium (CPC) with Kazakhstan and Oman. More recently, "the Anglo Dutch oil firm Shell bowed to pressure to let Gazprom gain control of $ 20 billion natural gas project in Sakhalin Island, shocking foreign investors" (see Gumbel (2006b) [42]).

Were Russian proved reserves artificially raised? This remains an open question.

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