• La PROBABILITA’ è una misura quantitativa del grado di fiducia del verificarsi di un certo EVENTO.

- Se siamo sicuri che l’evento avrà luogo gli assegniamo una probabilità del 100% e se siamo sicuri che l’evento non avrà luogo gli assegniamo una dello 0%.
- Agli altri eventi, quelli di cui non è sicuro né l’accadere né il non accadere, gli assegniamo invece probabilità intermedie tra 0 e 1. Se un evento ha una probabilità di 0.5 (50%) è altrettanto probabile che accada e che non accada.

![Diagram](image)

Il non accadere dell’evento è più probabile dell’accadere.

L’accadere dell’evento è più probabile del non accadere.

**Calcolo di P(Ci|A)***

**TEOREMA:** Sia A un evento, e C1, C2, ..., Ck una famiglia di eventi mutuamente esclusivi ed esaustivi. Allora

\[
P(Ci|A) = \frac{P(Ci) \cdot P(A|Ci)}{\sum P(Cj) \cdot P(A|Cj)}
\]

dove P(Ci|A) = probabilità a posteriori

e P(Ci) = probabilità a priori.

- La seconda espressione per P(Ci|A) nel Teorema è una conseguenza del Teorema delle Probabilità Totali: P(A) = \( \sum P(Cj) \cdot P(A|Cj) \).
- **ESEMPIO 39:** La probabilità che il paziente abbia mal di testa dato che ha problemi alla vista è

\[
P(C4|A) = \frac{0.20 \times 0.15}{0.2536} = 0.091
\]

poiché P(A) = 0.14 + 0.0002 + 0.069 + 0.03 + 0.0045 + 0.0099 = 0.2536

---

**TEOREMA DELLE CAUSE (BAYES)**

- Abbiamo visto che nel trattare le probabilità condizionate è spesso utile poter mettere in relazione P(B|A) con P(A|B). Ci viene allora in aiuto il risultato dato per primo dal Reverendo Thomas Bayes nel 1763: **Il Teorema delle Cause**.
- **ESEMPIO 39:** Un paziente si presenta dal medico accusando un forte mal di testa (A). Il medico sa che l’evento A (mal di testa) è l’effetto di k=6 possibili cause:

  - C1 = influenza
  - C2 = colpo di bastone dato dalla moglie
  - C3 = indigestione
  - C4 = problemi alla vista
  - C5 = insonnia
  - C6 = altre patologie specifiche

- Inoltre il medico sa, sulla base della sua esperienza, che:

  - P(A|C1) = 0.40 \( P(C1) = 0.35 \)
  - P(A|C2) = 0.02 \( P(C2) = 0.01 \)
  - P(A|C3) = 0.23 \( P(C3) = 0.30 \)
  - P(A|C4) = 0.15 \( P(C4) = 0.20 \)
  - P(A|C5) = 0.09 \( P(C5) = 0.05 \)
  - P(A|C6) = 0.11 \( P(C6) = 0.09 \)

Come valutare \( P(Ci|A) \)?
Judgement on Bayes

Bayes’ theorem should not be used in British courts unless the underlying statistics are “firm”. That is the apparent effect of the ruling of a UK judge in a recent court case. What the definition of “firm” statistics may be is not known.

The case is known as Rex v. T. In an appeal against a murder conviction, a shoeprint from a pair of Nike trainers, found at the scene of the crime, seemed to match a pair of shoes found at the home of the accused. An expert statistical witness gave evidence on the likelihood of this happening by chance. The expert, using Bayes’ theorem, made what the judge believed were poor calculations about the likelihood of the match. He also gave what the judge found to be a poor explanation of what he had done. The conviction was quashed. The judge also ruled against using similar statistical analysis in the courts in future.
Therefore the probability of someone with a positive mammogram having breast cancer is

\[
\frac{180,000}{3.5 \text{ million}} \approx 5\%.
\]

That means that 95% of the women who receive the horrible news that the mammogram has revealed something suspicious, and that they must return for further testing, are just fine. The mammogram led to them: Is a test worth this level of accuracy worth doing?

Without doubt, the answer to this question should be phrased in terms of human suffering and the length of time that lives are extended. I will get to that shortly. But to gain some understanding of at least one reason why the combination of the old standard of annual mammograms has been so vehemently supported, it is revealing to look into the amount of money at stake.

Mammograms cost between $100 and $200 each. Let us use the smaller figure. So for 33.5 million mammograms the cost (conservatively) is $3.35 billion. Next, the 3.5 million positive mammograms are redone, adding $350 million to the total. This will yield about 350,000 positives that will require a biopsy. A biopsy’s cost varies between about $1000 for a thin needle aspiration and $5000 for a fuller surgical procedure. Let us use the lower number. So to biopsy the 350,000 positive mammograms the cost would be at least $350 million. So far the cost is greater than $4 billion.

Breast cancer screening review

An independent review is to examine the merits of the NHS breast cancer screening programme in England. It follows disagreement over the effectiveness of the programme, how many lives it saves.

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Prospective first-trimester screening for trisomy 21 in 30,564 pregnancies

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Objective: This study was undertaken to evaluate the performance of a 1-stop clinic for first-trimester assessment of risk (OSCAR) for trisomy 21 by a combination of maternal age, fetal nuchal translucency (NT) thickness, and maternal serum-free \textit{β}-human chorionic gonadotrophin (hCG) and pregnancy-associated plasma protein-A (PAPP-A).

Study design: OSCAR was carried out in 30,564 pregnancies at 11 to 13+6 weeks. Patienc-specific risks for trisomy 21 and detection and false-positive rates were calculated.

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Variazione del rischio relativo per la sindrome di Down in relazione all’età materna a termine di gravidanza. Nell’asse orizzontale è rappresentata l’età e su quello verticale il numero di casi osservati per 1000 gravidanze.
### Table 1
Median (interquartile range) of maternal age in years, delta fetal NT thickness in millimeters, maternal serum free β-hCG and PAPP-A in multiples of the median (MoM) and estimated risk for trisomy 21 of 1 in 300 or greater in the chromosomally normal and abnormal pregnancies

<table>
<thead>
<tr>
<th>Fetal karyotype</th>
<th>N</th>
<th>Maternal age</th>
<th>Delta NT</th>
<th>β-hCG</th>
<th>PAPP-A</th>
<th>Estimated risk of ≥ 1 in 300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>30,236</td>
<td>34 (31-37)</td>
<td>0.0 (-0.2-0.3)</td>
<td>1.0 (0.7-1.6)</td>
<td>1.1 (0.7-1.5)</td>
<td>2,053 7.5 (7.2-7.8)</td>
</tr>
<tr>
<td>Trisomy 21</td>
<td>158</td>
<td>58 (55-61)*</td>
<td>1.0 (0.7-1.5)*</td>
<td>2.6 (1.6-2.8)*</td>
<td>0.5 (0.3-0.8)*</td>
<td>185 93.4 (85.0-96.1)</td>
</tr>
<tr>
<td>Trisomy 18</td>
<td>52</td>
<td>38 (37-40)*</td>
<td>3.0 (0.8-5.3)*</td>
<td>0.3 (0.1-0.4)*</td>
<td>0.1 (0.1-0.2)*</td>
<td>46 92.3 (81.9-97.0)</td>
</tr>
<tr>
<td>Trisomy 13</td>
<td>27</td>
<td>37 (34-38)*</td>
<td>1.7 (0.3-3.8)*</td>
<td>0.6 (0.3-0.8)*</td>
<td>0.3 (0.2-0.4)*</td>
<td>24 88.9 (72.0-96.1)</td>
</tr>
<tr>
<td>Turner syndrome</td>
<td>19</td>
<td>35 (31-37)*</td>
<td>7.1 (5.1-8.4)*</td>
<td>1.6 (0.7-1.5)*</td>
<td>0.4 (0.2-0.6)*</td>
<td>16 84.2 (62.6-94.6)</td>
</tr>
<tr>
<td>Other*</td>
<td>38</td>
<td>35.0 (34-39)</td>
<td>0.7 (-0.1-1.2)*</td>
<td>1.4 (0.2-2.2)*</td>
<td>0.6 (0.2-0.8)*</td>
<td>31 86.1 (71.4-93.9)</td>
</tr>
</tbody>
</table>

* P < .0001.
† P < .001. Comparison of each abnormal karyotype group to the normal group. † Not significant.
* Trisomies, deletions, partial trisomies, unbalanced translocations, see chromosome abnormalities.
† P < .05.
**Table II**

Detection rates for different false-positive rates in screening for trisomy 21, by the combination of maternal age, fetal NT, and maternal serum-free β-hCG and PAPP-A.

<table>
<thead>
<tr>
<th>Method of screening</th>
<th>Trisomy 21</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maternal age (MA)</td>
<td>196</td>
<td>22 (11.1%)</td>
<td>35 (17.9%)</td>
<td>62 (31.4%)</td>
<td>51 (26.0%)</td>
<td>61 (31.5%)</td>
<td>91 (46.4%)</td>
</tr>
<tr>
<td>MA and NT</td>
<td>196</td>
<td>123 (62.8%)</td>
<td>135 (68.8%)</td>
<td>154 (78.6%)</td>
<td>150 (81.6%)</td>
<td>171 (87.2%)</td>
<td></td>
</tr>
<tr>
<td>MA, NT, β-hCG, and PAPP-A</td>
<td>196</td>
<td>149 (76.0%)</td>
<td>156 (79.1%)</td>
<td>168 (80.7%)</td>
<td>173 (86.3%)</td>
<td>177 (90.3%)</td>
<td>184 (92.0%)</td>
</tr>
</tbody>
</table>

**Table III**

False-positive rates for different detection rates in screening for trisomy 21 by the combination of maternal age, fetal NT thickness, and maternal serum-free β-hCG and PAPP-A.

<table>
<thead>
<tr>
<th>Method of screening</th>
<th>Detection rate of Trisomy 21 pregnancies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5%</td>
</tr>
<tr>
<td>Maternal age (MA)</td>
<td>0.43%</td>
</tr>
<tr>
<td>MA and NT</td>
<td>0.44%</td>
</tr>
<tr>
<td>MA, NT, β-hCG, and PAPP-A</td>
<td>0.52%</td>
</tr>
</tbody>
</table>

* In our population there were 30,034 normal and 196 trisomy 21 pregnancies.

**Figure 2**

Relationship between detection rates and false-positive rates in screening for trisomy 21 by maternal age alone, maternal age and fetal NT, and by the combination of maternal age, fetal NT, and maternal serum biochemistry.
Figure 2.2.
Receiver operating characteristic (ROC) curve for the data in Table 2.6 and Figure 2.1.